ALTERNATING CURRENT

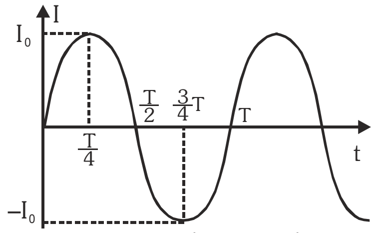
1. ALTERNATING CURRENT AND VOLTAGE

Voltage or current is said to be alternating if it changes continously in magnitude and perodically in direction. It can be represented by a sine curve or cosine curve

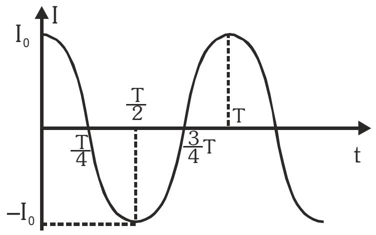
$$I = I_0 \sin \omega t \qquad \text{or} \qquad I = I_0 \cos \omega t$$

where I = Instantaneous value of current at time t, $\qquad I_0$ = Amplitude or peak value

ω = Angular frequency $\omega = \dfrac{2\pi}{T} = 2\pi f \qquad T$ = time period $\quad f$ = frequency



I as a sine function of t



I as a cosine function of t

1.1 Amplitude of AC

The maximum value of current in either direction is called peak value or the amplitude of current. It is represented by I_0. Peak to peak value = $2I_0$

1.2 Periodic Time

The time taken by alternating current to complete one cycle of variation is called periodic time or time period of the current.

1.3 Frequency

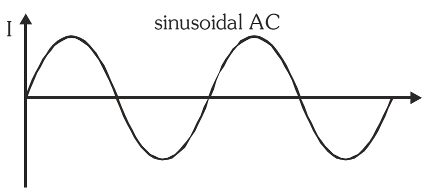
The number of cycle completed by an alternating current in one second is called the frequency of the current.

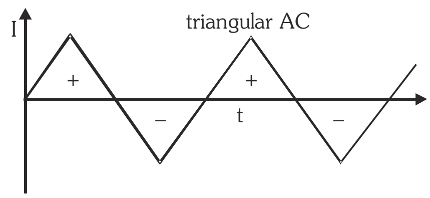
UNIT : (cycle/s) or (Hz)

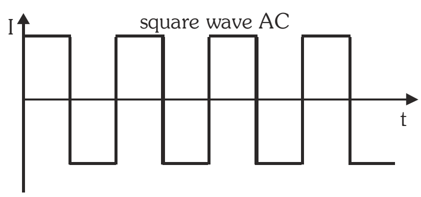
In India : f = 50 Hz , supply voltage = 220 volt $\qquad$ **In USA :** f = 60 Hz ,supply voltage = 110 volt

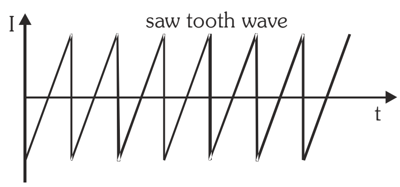
1.4 Condition required for current/ voltage to be Alternating

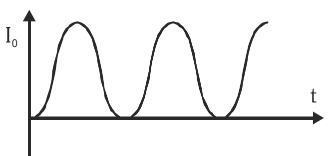
- Amplitude is constant.
- Alternate half cycle is positive and half negative.
- The alternating current continuously varies in magnitude and periodically reverses its direction.



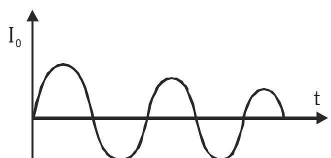




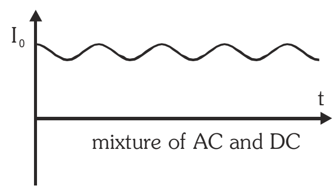




Not AC (direction not change)



Not AC (not periodic)



mixture of AC and DC

1.5 Average Value or Mean Value

The mean value of A.C over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit in the same time.

average value of current for half cycle $\langle I \rangle = \dfrac{\int_0^{T/2} I dt}{\int_0^{T/2} dt}$

Average value of $I = I_0 \sin \omega t$ over the positive half cycle :

$$I_{av} = \frac{\int_0^{\frac{T}{2}} I_0 \sin \omega t \, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2I_0}{\omega T}\left[-\cos \omega t\right]_0^{\frac{T}{2}} = \frac{2I_0}{\pi}$$

$< \sin\theta > = < \sin 2\theta >= 0$
$< \cos\theta >= < \cos 2\theta >= 0$
$< \sin\theta \cos\theta > = 0$
$< \sin^2\theta > = < \cos^2\theta >= \frac{1}{2}$

- For symmetric AC, average value over full cycle = 0,

  Average value of sinusoidal AC

Full cycle	(+ve) half cycle	(–ve) half cycle
0	$\frac{2I_0}{\pi}$	$\frac{-2I_0}{\pi}$

As the average value of AC over a complete cycle is zero, it is always defined over a half cycle which must be either positive or negative

1.6 Maximum Value

- $I = a \sin\theta \Rightarrow I_{Max.} = a$
- $I = a + b \sin\theta \Rightarrow I_{Max.} = a + b$ (if a and b > 0)
- $I = a \sin\theta + b \cos\theta \Rightarrow I_{Max.} = \sqrt{a^2 + b^2}$
- $I = a \sin^2\theta \Rightarrow I_{Max.} = a \ (a > 0)$

1.7 Root Mean square (rms) Value

It is value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}}$$ rms value = virtual value = apparent value

rms value of $I = I_0 \sin \omega t$:

$$I_{rms} = \sqrt{\frac{\int_0^T (I_0 \sin \omega t)^2 dt}{\int_0^T dt}} = \sqrt{\frac{I_0^2}{T}\int_0^T \sin^2 \omega t \, dt} = I_0\sqrt{\frac{1}{T}\int_0^T\left[\frac{1-\cos 2\omega t}{2}\right]dt} == I_0\sqrt{\frac{1}{T}\left[\frac{t}{2} - \frac{\sin 2\omega t}{2 \times 2\omega}\right]_0^T} = \frac{I_0}{\sqrt{2}}$$

- If nothing is mentioned then values printed in a.c circuit on electrical appliances, any given or unknown values, reading of AC meters are assumed to be RMS.

Current	**Average**	**Peak**	**RMS**	**Angular fequency**
$I_1 = I_0 \sin \omega t$	0	I_0	$\frac{I_0}{\sqrt{2}}$	ω
$I_2 = I_0 \sin \omega t \cos \omega t = \frac{I_0}{2}\sin 2\omega t$	0	$\frac{I_0}{2}$	$\frac{I_0}{2\sqrt{2}}$	2ω
$I_3 = I_0 \sin\omega t + I_0 \cos\omega t$	0	$\sqrt{2} I_0$	I_0	ω

- For above varieties of current

$$\boxed{\text{rms} = \frac{\text{Peak value}}{\sqrt{2}}}$$

1.8 Measurement of A.C.

Alternating current and voltages are measured by a.c. ammeter and a.c. voltmeter respectively. Working of these instruments is based on heating effect of current, hence they are also called hot wire instruments.

Terms	D.C. meter	A.C. meter
Name	moving coil instrument	hot wire instrument
Based on	magnetic effect of current	heating effect of current
Reads	average value	r.m.s. value
If used in	A.C. circuit then they reads zero	A.C. or D.C. then meter works
	$\because$ average value of A.C. = zero	properly as it measures rms value
Deflection	deflection $\propto$ current	deflection $\propto$ heat
	$\phi \propto I$ (linear)	$\phi \propto I^2_{rms}$ (non linear)
Scale	Uniform Seperation	Non uniform sepration
ϕ = Number	I - 1 2 3 4 5	I - 1 2 3 4 5
of divisions	ϕ - 1 2 3 4 5	ϕ - 1 4 9 16 25

1.9 Phase and phase difference

(a) Phase

$I = I_0 \sin(\omega t \pm \phi)$

Initial phase = ϕ (it does not change with time)

Instantaneous phase = $\omega t \pm \phi$ (it changes with time)

- Phase decides both value and sign.
- **UNIT:** radian

(b) Phase difference

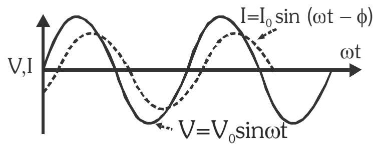
Voltage $V = V_0 \sin(\omega t + \phi_1)$

Current $I = I_0 \sin(\omega t + \phi_2)$

- Phase difference of I w.r.t. V $\quad \phi = \phi_2 - \phi_1$
- Phase difference of V w.r.t. I $\quad \phi = \phi_1 - \phi_2$

1.10 Lagging and leading Concept

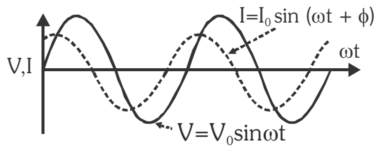
(a) V leads I or I lags V $\rightarrow$ It means, V reach maximum before I



Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin(\omega t - \phi)$

and if $V = V_0 \sin(\omega t + \phi)$ then $I = I_0 \sin \omega t$

(b) V lags I or I leads V $\rightarrow$ It means V reach maximum after I



Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin(\omega t + \phi)$

and if $V = V_0 \sin(\omega t - \phi)$ then $I = I_0 \sin \omega t$

1.11 Phasor and Phasor diagram

A diagram representing alternating current and voltage (of same frequency) as vectors (phasor) with the phase angle between them is called phasor diagram.

Let $V = V_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$

In figure (a) two arrows represents phasors. The length of phasors represents the maximum value of quantity. The projection of a phasor on y-axis represents the instantaneous value of quantity. In figure (b) two arrows represents phasor. Their length represents maximum value.

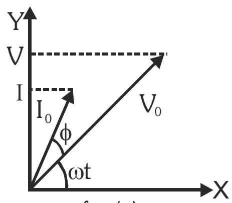


fig (a)

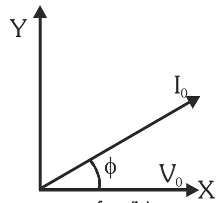


fig (b)

1.12 Advantages of AC

- A.C. is cheaper than D.C
- It can be easily converted into D.C. (by rectifier)
- It can be controlled easily (choke coil)
- It can be transmitted over long distance at low power loss.
- It can be stepped up or stepped down with the help of transformer.

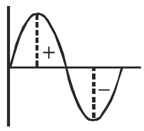
GOLDEN KEY POINTS

- AC can't be used in

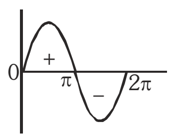
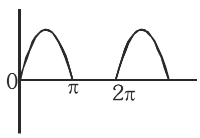
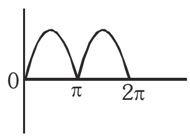
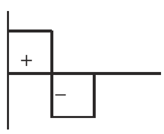
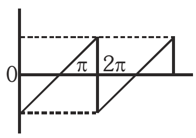
  (a) Charging of battery or capacitor (as its average value = 0)

  (b) Electrolysis and electroplating (Due to large inertia, ions can not follow frequency of A.C)

- The rate of change of A.C. →
  - Minimum, at that instant when they are near their peak values
  - Maximum, at that instant when they change their direction.
- For alternating current $I_0 > I_{rms} > I_{av.}$
- Average value over half cycle is zero if one quarter is positive and the other quarter is negative.



- Average value of symmetrical AC for a cycle is zero thats why average potential difference on any element in A.C circuit is zero.
- The instrument based on heating effect of current are works on both A.C and D.C supply and also provides same heating for same value of A.C (rms) and D.C. that's why a bulb bright equally in D.C. and A.C. of same value.
- If the frequency of AC is f then it becomes zero, 2f times in one second and the direction of current changes 2f times in one second. Also it become maximum 2f times in one second.
- Some Important wave forms and their RMS and Average Value

Nature of wave form	Wave-form	RMS Value	Average or mean Value
Sinusoidal		$\frac{I_0}{\sqrt{2}} = 0.707\ I_0$	$\frac{2I_0}{\pi} = 0.637\ I_0$ (Half)
Half wave rectifier		$\frac{I_0}{2} = 0.5\ I_0$	$\frac{I_0}{\pi} = 0.318\ I_0$ (Full)
Full wave rectifier		$\frac{I_0}{\sqrt{2}} = 0.707\ I_0$	$\frac{2I_0}{\pi} = 0.637\ I_0$ (Half and Full)
Square or Rectangular		I_0	I_0 (Half)
Saw Tooth wave		$\frac{I_0}{\sqrt{3}}$	$\frac{I_0}{2}$ (Half)

- D.C meter in AC circuit reads zero because < AC > = 0 (for complete cycle)
- AC meter works in both AC and DC

Illustration 1.

If I = 2 $\sqrt{t}$ ampere then calculate average and rms values over t = 2 to 4 s

Solution

$$\langle I\rangle = \frac{\int_2^4 2\sqrt{t}.dt}{\int_2^4 dt} = \frac{4}{3}\frac{(t^{\frac{3}{2}})_2^4}{(t)_2^4} = \frac{2}{3}\left[8-2\sqrt{2}\right] \text{ and } I_{rms} = \sqrt{\frac{\int_2^4 (2\sqrt{t})^2 dt}{\int_2^4 dt}} = \sqrt{\frac{\int_2^4 4t\,dt}{2}} = \sqrt{2\left[\frac{t^2}{2}\right]_2^4} = 2\sqrt{3}\,A$$

Illustration 2.

If E = 20 sin (100π t) volt then calculate value of E at t = $\frac{1}{600}$ s

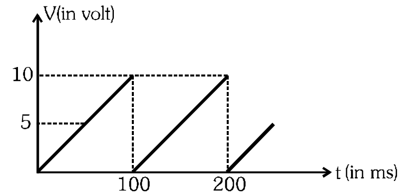
Solution

$$\text{At } t = \frac{1}{600}\text{ s} \quad E = 20 \text{ Sin}\left[100\pi \times \frac{1}{600}\right] = 20 \sin\left[\frac{\pi}{6}\right] = 20 \times \frac{1}{2} = 10V$$

Illustration 3.

A periodic voltage wave form has been shown in figure. Determine.

(a) Frequency of the wave form. (b) Average value.



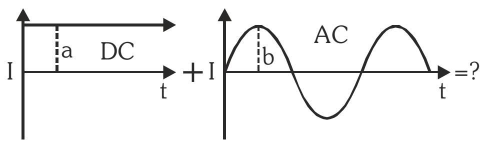
Solution

(a) After 100 ms wave is repeated so time period is T = 100 ms. ⇒ $f = \frac{1}{T} = 10$ Hz

(b) Average value = Area/time period $= \frac{(1/2)\times 100 \times 10}{(100)} = 5$ volt

Illustration 4.

If a direct current of value a ampere is superimposed on an alternating current I = b sinωt flowing through a wire, what is the effective value of the resulting current in the circuit ?



Solution

As current at any instant in the circuit will be $I = I_{DC} + I_{AC} = a + b\sin\omega t$

$$\therefore I_{eff} = \sqrt{\frac{1}{T}\int_0^T I^2 dt} = \sqrt{\frac{1}{T}\int_0^T (a+b\sin\omega t)^2 dt} = \sqrt{\frac{1}{T}\int_0^T (a^2 + 2ab\sin\omega t + b^2\sin^2\omega t)dt}$$

but as $\frac{1}{T}\int_0^T \sin\omega t dt = 0$ and $\frac{1}{T}\int_0^T \sin^2\omega t dt = \frac{1}{2}$ $\quad \therefore I_{eff} = \sqrt{a^2 + \frac{1}{2}b^2}$

Illustration 5.

The Equation of current in AC circuit is $I = 4\sin\left[100\pi t + \frac{\pi}{3}\right]$A. Calculate.

(i) RMS Value (ii) Peak Value (iii) Frequency (iv) Initial phase (v) Current at t = 0

Solution

(i) $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ A

(ii) Peak value $I_0 = 4$A

(iii) $\because$ $\omega = 100\ \pi$ rad/s $\therefore$ frequency $f = \frac{100\pi}{2\pi} = 50$ Hz

(iv) Initial phase $= \frac{\pi}{3}$

(v) At t = 0, $I = 4\sin\left[100\pi \times 0 + \frac{\pi}{3}\right] = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$ A

Illustration 6.

If $I = I_0 \sin \omega t$, $E = E_0 \cos\left[\omega t + \frac{\pi}{3}\right]$. Calculate phase difference between E and I

Solution

$I = I_0 \sin \omega t$ and $E = E_0 \sin\left[\frac{\pi}{2} + \omega t + \frac{\pi}{3}\right]$ $\therefore$ phase difference $= \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

Illustration 7.

If E = 500 sin (100 πt) volt then calculate time taken to reach from zero to maximum.

Solution

$\because \omega = 100\ \pi \Rightarrow T = \frac{2\pi}{100\pi} = \frac{1}{50}$ s, time taken to reach from zero to maximum $= \frac{T}{4} = \frac{1}{200}$ s

Illustration 8.

If Phase Difference between E and I is $\frac{\pi}{4}$ and f = 50 Hz then calculate time difference.

Solution

$\because$ $2\pi \equiv T \therefore \frac{\text{Phase difference}}{2\pi} = \frac{\text{time difference}}{T}$

$\Rightarrow$ Time difference $= \frac{T}{2\pi} \times \frac{\pi}{4} = \frac{T}{8} = \frac{1}{50 \times 8} = 2.5$ms

BEGINNER'S BOX-1

1. Explain why A.C. is more dangerous than D.C. ?

2. Show that average heat produced during a cycle of AC is same as produced by DC with $I = I_{rms}$.

3. An ordinary moving coil ammeter used for d.c., cannot be used to measure a.c. even if its frequency is low why ?

4. Find the time required for a 50Hz alternating current to change its value from zero to rms value.

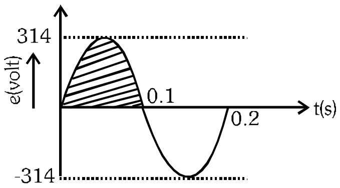
5. The current and voltage in a circuit is given by

$i = 3.5 \sin (628t + 30°)$ A , $V = 28 \sin (628t - 30°)$ volt. Find

(a) time period of current

(b) phase difference between voltage and current.

6. The figure given below shows the variation of an alternating emf with time. What is the average value of the emf for the shaded part of the graph?



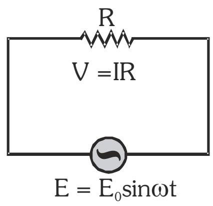
2. DIFFERENT TYPES OF AC CIRCUITS

In order to study the behaviour of A.C. circuits we classify them into two categories :

(a) Simple circuits containing only one basic element i.e. resistor (R) or inductor (L) or capacitor (C) only.

(b) Complicated circuit containing any two of the three circuit elements R, L and C or all of the three elements.

2.1 AC circuit containing pure resistance

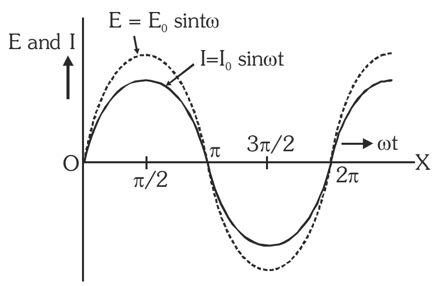


Let at any instant t, the current in the circuit = I.

Potential difference across the resistance = I R

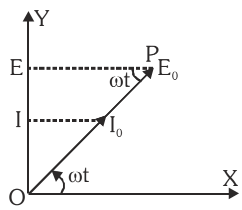
with the help of kirchoff's circuital law $E - IR = 0 \Rightarrow E_0 \sin \omega t = IR$

$\Rightarrow I = \dfrac{E_0}{R}\sin\omega t = I_0 \sin\omega t$ ($I_0 = \dfrac{E_0}{R}$ = peak or maximum value of current)



Alternating current developed in a pure resistance is also of the sinusoidal nature. In a.c. circuits containing pure resistance, the voltage and current are in the same phase. The vector or phasor diagram which represents the phase relationship between alternating current and alternating e.m.f. are as shown in figure.

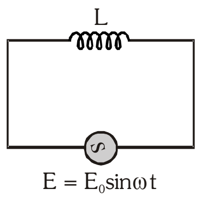
In the a.c. circuit having R only, as current and voltage are in the same phase, hence in fig. both phasors E_0 and I_0 are in the same direction, making an angle ωt with OX. Their projections on Y-axis represent the instantaneous values of alternating current and voltage.



i.e. $I = I_0 \sin\omega t$ and $E = E_0 \sin\omega t$.

Since $I_0 = \dfrac{E_0}{R}$, hence $\dfrac{I_0}{\sqrt{2}} = \dfrac{E_0}{R\sqrt{2}} \Rightarrow I_{rms} = \dfrac{E_{rms}}{R}$

2.2 AC circuit containing pure inductance



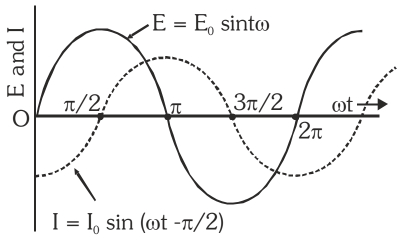
A circuit containing a pure inductance L (having zero ohmic resistance) connected with a source of alternating emf. Let the alternating e.m.f. $E = E_0 \sin \omega t$

When a.c. flows through the circuit, emf induced across inductance $= -L\dfrac{dI}{dt}$

Note : Negative sign indicates that induced emf acts in opposite direction to that of applied emf.

Because there is no other circuit element present in the circuit other than inductance so with the help of Kirchoff's circuital law $E + \left(-L\dfrac{dI}{dt}\right) = 0 \Rightarrow E = L\dfrac{dI}{dt}$ so we get $I = \dfrac{E_0}{\omega L}\sin\left(\omega t - \dfrac{\pi}{2}\right)$

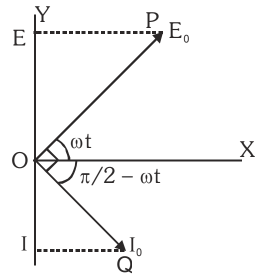
Maximum current $I_0 = \dfrac{E_0}{\omega L} \times 1 = \dfrac{E_0}{\omega L}$, Hence, $I = I_0 \sin\left(\omega t - \dfrac{\pi}{2}\right)$



In a pure inductive circuit current always lags behind the emf by $\dfrac{\pi}{2}$

or alternating emf leads the a. c. by a phase angle of $\dfrac{\pi}{2}$.

Expression $I_0 = \dfrac{E_0}{\omega L}$ resembles the expression $\dfrac{E}{I} = R$.



This non-resistive opposition to the flow of A.C. in a circuit is called the inductive reactance (X_L) of the circiut.

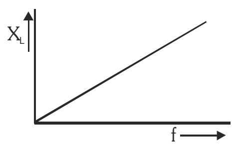
$X_L = \omega L = 2\pi f L$ where f = frequency of A.C.

Unit of X_L : ohm

(ωL) = Unit of L × Unit of ($\omega = 2\pi f$) = henry × sec^{-1}

$$= \frac{\text{volt}}{\text{ampere / sec}} \times \text{sec}^{-1} = \frac{\text{volt}}{\text{ampere}} = \text{ohm}$$

Inductive reactance $X_L \propto f$

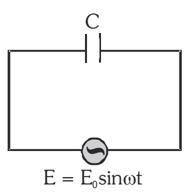


Higher the frequency of A.C. higher is the inductive reactance offered by an inductor in an A.C. circuit.

For d.c. circuit, f = 0 $\therefore X_L = \omega L = 2\pi f L = 0$

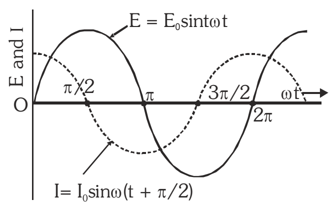
Hence, inductor offers no opposition to the flow of d.c. where as a resistive path to a.c.

2.3 AC circuit containing pure capacitance



A circuit containing an ideal capacitor of capacitance C connected with a source of alternating emf as shown in fig. The alternating e.m.f. in the circuit $E = E_0 \sin \omega t$. When alternating e.m.f. is applied across the capacitor a similarly varying alternating current flows in the circuit.

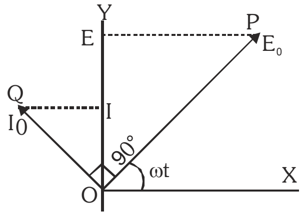
The two plates of the capacitor become alternately positively and negatively charged and the magnitude of the charge on the plates of the capacitor varies sinusoidally with time. Also the electric field between the plates of the capacitor varies sinusoidally with time.Let at any instant t charge on the capacitor = q



Instantaneous potential difference across the capacitor E = q/C

$\Rightarrow q = C E \Rightarrow q = CE_0 \sin \omega t$

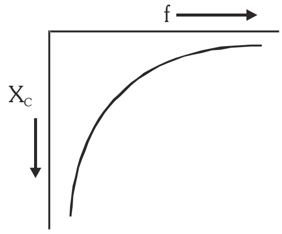
The instantaneous value of current $I = \frac{dq}{dt} = \frac{d}{dt}\left(CE_0 \sin \omega t\right) = CE_0 \omega \cos \omega t$



$$\Rightarrow I = \frac{E_0}{\left(1/\omega C\right)} \sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \text{ where } I_0 = \omega CE_0$$

In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of $\pi/2$. The alternating emf lags behinds the alternating current by a phase angle of $\pi/2$.

IMPORTANT POINTS



E/I is the resistance R when both E and I are in phase, in present case they differ in phase by $\frac{\pi}{2}$, hence $\frac{1}{\omega C}$ is not the resistance of the capacitor, the capacitor offer opposition to the flow of A.C. This non-resistive opposition to the flow of A.C. in a pure capacitive circuit is known as capacitive reactance X_C.

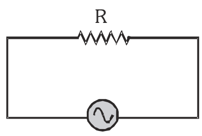
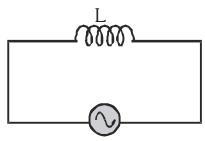
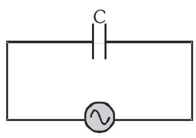
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

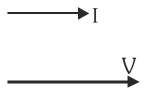
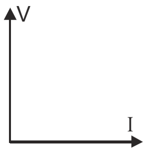
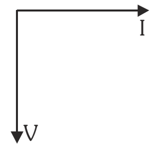
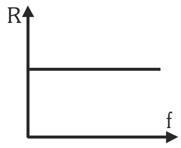
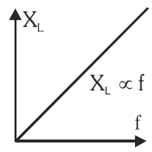
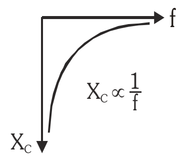
Unit of X_C : ohm

Capacitive reactance X_C is inversely proportional to frequency of A.C. X_C decreases as the frequency increases.

For d.c. circuit f = 0 $\quad \therefore X_C = \frac{1}{2\pi f C} = \infty$ but has a very small value for a.c.

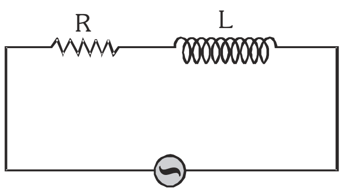
This shows that capacitor blocks the flow of d.c. but provides an easy path for a.c.

INDIVIDUAL COMPONENTS (R or L or C)			
TERM	**R**	**L**	**C**
Circuit			
Supply Voltage	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$
Current	$I = I_0 \sin \omega t$	$I = I_0 \sin (\omega t - \frac{\pi}{2})$	$I = I_0 \sin (\omega t + \frac{\pi}{2})$
Peak Current	$I_0 = \frac{V_0}{R}$	$I_0 = \frac{V_0}{\omega L}$	$I_0 = \frac{V_0}{1/\omega C} = V_0 \omega C$
Impedance (Ω)	$\frac{V_0}{I_0} = R$	$\frac{V_0}{I_0} = \omega L = X_L$	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
$Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$	R = Resistance	X_L=Inductive reactance.	X_C=Capacitive reactance.

Phase difference	zero (in same phase)	$+\frac{\pi}{2}$ (V leads I)	$-\frac{\pi}{2}$ (V lags I)
Phasor diagram			
Variation of Z with *f*			
G, S_L, S_C (mho, seiman)	G=1/R = conductance.	$S_L = 1/X_L$ Inductive susceptance	$S_C = 1/X_C$ Capacitive susceptance
Behaviour of device in D.C. and A.C	Same in A C and D C	L passes DC easily (because $X_L = 0$) while gives a high impedance for the A.C. of high frequency ($X_L \propto f$)	C - blocks DC (because $X_C = \infty$) while provides an easy path for the A.C. of high frequency $\left[X_C \propto \frac{1}{f}\right]$
Ohm's law	$V_R = IR$	$V_L = IX_L$	$V_C = IX_C$

2.4 Resistance and inductance in series (R-L circuit)

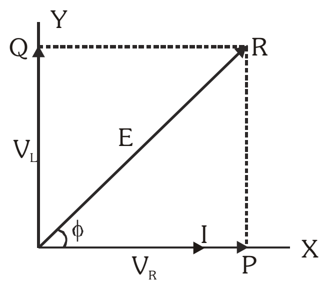
A circuit containing a series combination of a resistance R and an inductance L, connected with a source of alternating e.m.f. E as shown in figure.



$E = E_0 \sin\omega t$

- **phasor diagram For L-R circuit**

Let in a L-R series circuit, applied alternating emf is $E = E_0 \sin\omega t$. As R and L are joined in series, hence current flowing through both will be same at each instant. Let I be the current in the circuit at any instant and V_L and V_R the potential differences across L and R respectively at that instant.



Then $V_L = IX_L$ and $V_R = IR$

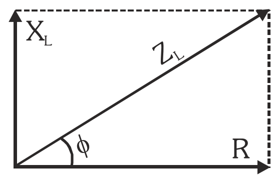
Now, V_R is in phase with the current while V_L leads the current by $\frac{\pi}{2}$.

So V_R and V_L are mutually perpendicular (Note : $E \neq V_R + V_L$)

The vector OP represents V_R (which is in phase with I), while OQ represents V_L (which leads I by 90°).

The resultant of V_R and V_L = the magnitude of vector OR $E = \sqrt{V_R^2 + V_L^2}$

Thus $E^2 = V_R{}^2 + V_L{}^2 = I^2 (R^2 + X_L^2) \Rightarrow I = \dfrac{E}{\sqrt{R^2 + X_L^2}}$



The phasor diagram shown in fig. also shows that in L-R circuit the applied emf E leads the current I or conversely the current I lags behind the e.m.f. E by a phase angle ϕ

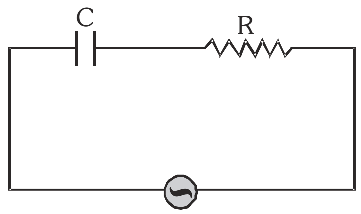
$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R} \quad \Rightarrow \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

- **Inductive Impedance Z_L :**

In L-R circuit the maximum value of current $I_0 = \dfrac{E_0}{\sqrt{R^2 + \omega^2 L^2}}$ Here $\sqrt{R^2 + \omega^2 L^2}$ represents the effective opposition offered by L-R circuit to the flow of a.c. through it. It is known as impedance of L-R circuit and is represented by Z_L. $Z_L = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi f L)^2}$

The reciprocal of impedance is called admittance $Y_L = \dfrac{1}{Z_L} = \dfrac{1}{\sqrt{R^2 + \omega^2 L^2}}$

2.5 Resistance and capacitor in series (R-C circuit)



A circuit containing a series combination of a resistance R and a capacitor C, connected with a source of e.m.f. of peak value E_0 as shown in fig.

- **phasor diagram For R-C circuit**

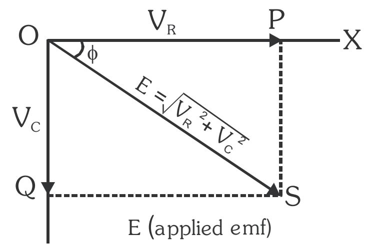
Current through both the resistance and capacitor will be same at every instant and the instantaneous potential differences across C and R are

$$V_C = I X_C \quad \text{and} \quad V_R = I R$$

where X_C = capacitive reactance and I = instantaneous current.

Now, V_R is in phase with I, while V_C lags behind I by 90°.

The phasor diagram is shown in fig.



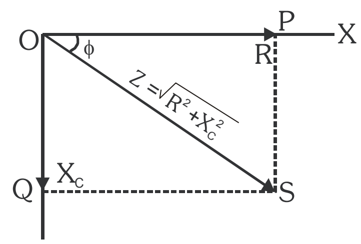
The vector OP represents V_R (which is in phase with I)

and the vector OQ represents V_C (which lags behind I by $\frac{\pi}{2}$).

The vector OS represents the resultant of V_R and V_C = the applied e.m.f. E.

Hence $V_R{}^2 + V_C{}^2 = E^2 \Rightarrow E = \sqrt{V_R^2 + V_C^2} \Rightarrow E^2 = I^2 (R^2 + X_C^2) \Rightarrow I = \dfrac{E}{\sqrt{R^2 + X_C^2}}$

The term $\sqrt{(R^2 + X_C^2)}$ represents the effective resistance of the R-C circuit and called the capacitive impedance Z_C of the circuit.



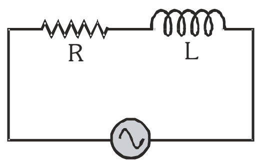
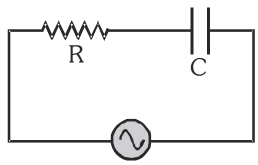
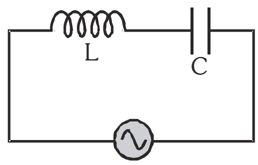
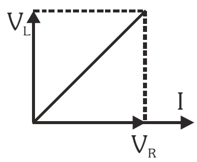
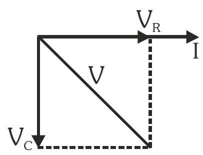
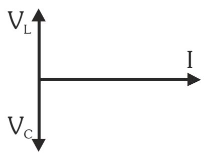
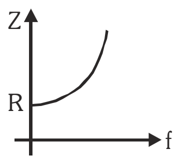
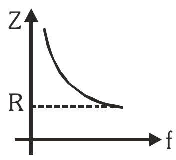
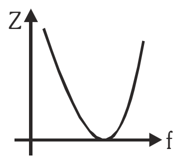
Hence, in C-R circuit $Z_C = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$

- **Capacitive Impedance Z_C :**

  In R-C circuit the term $\sqrt{R^2+X_C^2}$ effective opposition offered by R-C circuit to the flow of a.c. through it. It is known as impedance of R-C circuit and is represented by Z_C.The phasor diagram also shows that in R-C circuit the applied e.m.f. lags behind the current I (or the current I leads the emf E) by a phase angle ϕ given by

$$\tan\phi=\frac{V_C}{V_R}=\frac{X_C}{R}=\frac{1/\omega C}{R}=\frac{1}{\omega CR},\quad \tan\ \phi=\frac{X_C}{R}=\frac{1}{\omega CR}\ \Rightarrow\ \phi=\tan^{-1}\left(\frac{1}{\omega CR}\right)$$

2.6 Combination of components (R-L or R-C or L-C)

TERM	R-L	R-C	L-C
Circuit			
	I is same in R & L	I is same in R & C	I is same in L & C
Phasor diagram			
	$V^2 = V_R^2+V_L^2$	$V^2 = V_R^2+V_C^2$	$V = V_L - V_C\ (V_L>V_C)$ $V = V_C - V_L\ (V_C>V_L)$
Phase difference	V leads I ($\phi = 0$ to $\frac{\pi}{2}$)	V lags I ($\phi = -\frac{\pi}{2}$ to 0)	V lags I ($\phi = -\frac{\pi}{2}$, if $X_C>X_L$)
in between V & I			V leads I ($\phi = +\frac{\pi}{2}$, if $X_L>X_C$)
Impedance	$Z=\sqrt{R^2+X_L^2}$	$Z = \sqrt{R^2+(X_C)^2}$	$Z = \lvert X_L - X_C\rvert$
Variation of Z	as f $\uparrow$, Z $\uparrow$	as f $\uparrow$, Z $\downarrow$	as f $\uparrow$, Z first $\downarrow$ then $\uparrow$
with f			
At very low f	$Z \simeq R\ (X_L \to 0)$	$Z \simeq X_C$	$Z \simeq X_C$
At very high f	$Z \simeq X_L$	$Z \simeq R\ (X_C \to 0)$	$Z \simeq X_L$

GOLDEN KEY POINTS

- Phase diference between capacitive and inductive reactance is π
- Inductor is called Low pass filter because it allows low frequency signal to pass.
- Capacitor is called high pass filter because it allows high frequency signal to pass.

Illustration 9.

What is the inductive reactance of a coil if the current through it is 20 mA and voltage across it is 100 V.

Solution

$$\because \; V_L = IX_L \qquad \therefore \; X_L = \frac{V_L}{I} = \frac{100}{20\times10^{-3}} = 5\ k\Omega$$

Illustration 10.

The reactance of capacitor is 20 ohm. What does it mean? What will be its reactance if frequency of AC is doubled? What will be its, reactance when connected in DC circuit? What is its consequence?

Solution

The reactance of capacitor is 20 ohm. It means that the hinderance offered by it to the flow of AC at a specific frequency is equivalent to a resistance of 20 ohm. The reactance of capacitance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

Therefore by doubling frequency, the reactance is halved i.e., it becomes 10 ohm. In DC circuit f = 0. Therefore reactance of capacitor = ∞ (infinite). Hence the capacitor can not be used to control DC.

Illustration 11.

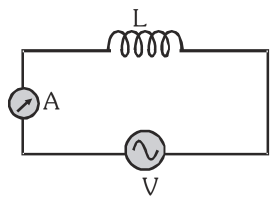
A capacitor of 50 pF is connected to an a.c. source of frequency 1kHz. Calculate its reactance.

Solution

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\times10^3\times50\times10^{-12}} = \frac{10^7}{\pi}\ \Omega$$

Illustration 12.

In given circuit applied voltage $V = 50\sqrt{2}$ sin $(100\pi t)$ volt and ammeter reading is 2A then calculate value of L



Solution

$V_{rms} = I_{rms} X_L \qquad \because$ Reading of ammeter $= I_{rms}$

$$X_L = \frac{V_{rms}}{I_{rms}} = \frac{V_0}{\sqrt{2}\,I_{rms}} = \frac{50\sqrt{2}}{\sqrt{2}\times2} = 25\ \Omega \Rightarrow L = \frac{X_L}{\omega} = \frac{25}{100\pi} = \frac{1}{4\pi}\ H$$

Illustration 13.

A 50 W, 100 V lamp is to be connected to an AC mains of 200 V, 50 Hz. What capacitance is essential to be put in series with the lamp ?

Solution

$\because$ resistance of the lamp $R = \frac{V_s^2}{W} = \frac{(100)^2}{50} = 200\ \Omega$ and the maximum current $I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}A$

$\therefore$ when the lamp is put in series with a capacitance and run at 200 V AC, from V = IZ

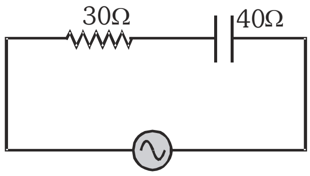
$$Z = \frac{V}{I} = \frac{200}{\frac{1}{2}} = 400\Omega \qquad \text{Now as in case of C–R circuit } Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}},$$

$$\Rightarrow R^2 + \frac{1}{(\omega C)^2} = (400)^2 \Rightarrow \frac{1}{(\omega C)^2} = 16\times10^4 - (200)^2 = 12\times10^4 \Rightarrow \frac{1}{\omega C} = \sqrt{12}\times10^2$$

$$\Rightarrow C = \frac{1}{100\pi\times\sqrt{12}\times10^2}F = \frac{100}{\pi\sqrt{12}}\mu F = 9.2\mu F$$

Illustration 14.

Calculate the impedance of the circuit shown in the figure.



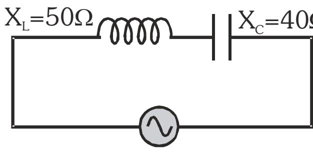
Solution

$$Z = \sqrt{R^2 + (X_c)^2} = \sqrt{(30)^2 + (40)^2} = \sqrt{2500} = 50\ \Omega$$

Illustration 15.

If $X_L = 50\ \Omega$ and $X_C = 40\ \Omega$. Calculate effective value of current in given circuit.

Solution



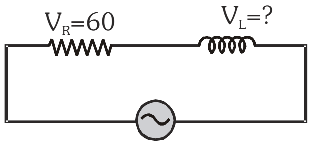
V=40sin 100π volt

$$Z = X_L - X_C = 10\ \Omega$$

$$I_0 = \frac{V_0}{Z} = \frac{40}{10} = 4\ A \Rightarrow \quad I_{rms} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\ A$$

Illustration 16.

In given circuit calculate, voltage across inductor



V=100√2 sinωt volt

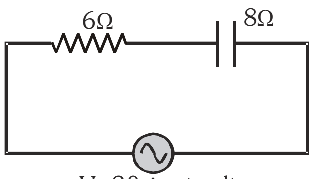
Solution

$$\because V^2 = V_R^2 + V_L^2 \qquad \therefore V_L^{\,2} = V^2 - V_R^{\,2}$$

$$V_L = \sqrt{V^2 - V_R^2} = \sqrt{(100)^2 - (60)^2} = \sqrt{6400} = 80\ V$$

Illustration 17.

In given circuit find out (i) impedance of circuit (ii) current in circuit



V=20sinωt volt

Solution

(i) $Z = \sqrt{R^2 + X_C^2} = \sqrt{(6)^2 + (8)^2} = 10\ \Omega$

(ii) $V = IZ \Rightarrow I = \frac{V_0}{Z} = \frac{20}{10} = 2A$, so $I_{rms} = \frac{2}{\sqrt{2}} = \sqrt{2}\,A$

Illustration 18.

When 10V, DC is applied across a coil current through it is 2.5 A, if 10V, 50 Hz A.C. is applied current reduces to 2 A. Calculate reactance of the coil.

Solution

For 10 V D.C. $\because V = IR$ $\qquad \therefore$ Resistance of coil $R = \frac{10}{2.5} = 4\Omega$

For 10 V A.C. $\leftarrow V = I\,Z \Rightarrow Z = \frac{V}{I} = \frac{20}{10} = 5\Omega$

$$\because Z = \sqrt{R^2 + X_L^2} = 5 \Rightarrow R^2 + X_L^2 = 25 \Rightarrow X_L^2 = 5^2 - 4^2 \Rightarrow X_L = 3\ \Omega$$

Illustration 19.

When an alternating voltage of 220V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit but it leads the applied voltage by $\pi/2$ radians.

(a) Name the devices X and Y.

(b) Calculate the current flowing in the circuit when same voltage is applied across the series combination of X and Y.

Solution

(a) X is resistor and Y is a capacitor

(b) Since the current in the two devices is the same (0.5A at 220 volt)
When R and C are in series across the same voltage then

$$R = X_C = \frac{220}{0.5} = 440\ \Omega \Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}} = \frac{220}{\sqrt{(440)^2 + (440)^2}} = \frac{220}{440\sqrt{2}} = 0.35A$$

BEGINNER'S BOX-2

1. A voltage V = 60 sin πt volt is applied across a 20 Ω resistor. What will an ac ammeter in series with the resistor read ?

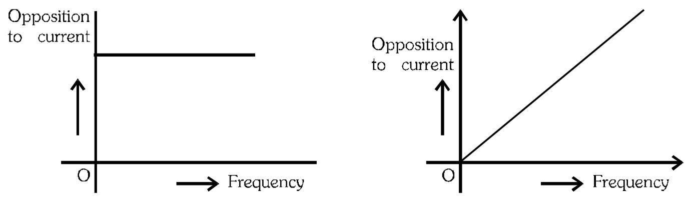
2. An alternating current source E = 100 sin (1000t) volt is connected through a inductor of 10 μH then write down the equation of current.

3. An alternating voltage E = 200 $\sqrt{2}$ sin (100t) volt is applied to 2H inductor through an a.c. ammeter. What will be reading of the ammeter ?

4. A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?

5. A 60 μF capacitor is connected to a 110 V. 60 Hz a.c. supply. Determine the rms value of the current in the circuit.

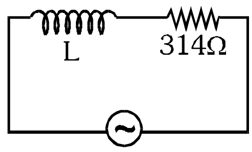
6. The given graphs (a) and (b) represent the variation of the opposition offered by the circuit element to the flow of alternating current, with frequency of the applied emf. Identify the circuit element corresponding to each graph.



7. When a series combination of inductance and resistance are connected with a 10V, 50 Hz a.c. source, a current of 1A flows in the circuit. The voltage leads the current by a phase angle of $\frac{\pi}{3}$ radian. Calculate the values of resistance and inductive reactance.

8. Calculate the impedance of a condenser in order to run a bulb rated at 10 watt 60 volt when connected in series to an A.C. source of 100 volt .

9. The current in the shown circuit is found to be $4\sin\left(314t-\frac{\pi}{4}\right)$A. Find the value of inductance.



10. A current of 4A flows in a coil when connected to a 12V dc source. If the same coil is connected to a 12V, 50 rad/s a.c. source, a current of 2.4A flows in the circuit. Determine the inductance of the coil.

11. An alternating voltage E=200 sin (300t) volt is applied across a series combination of R=10Ω and inductance of 800mH. Calculate the impedance of the circuit.

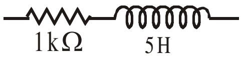
12. A coil of reactance 100 Ω and resistance 100 Ω is connected to a 240 V, 50 Hz a.c. supply.

(a) What is the maximum current in the coil ?

(b) What is the time lag between the voltage maximum and the current maximum ?

13. An inductance has a resistance of 100 Ω. When a.c. signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. Calculate the self inductance of the coil.

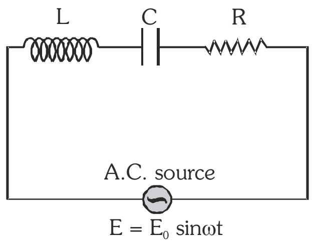
14. Match the following options –

Circuit component across an ac source (ω = 200 rad/sec)		Phase difference between current and source voltage	
(A)		(p)	$\frac{\pi}{2}$
(B)		(q)	$\frac{\pi}{6}$
(C)		(r)	$\frac{\pi}{4}$
(D)		(s)	$\frac{\pi}{3}$
(E)		(t)	None of the above

3. INDUCTANCE, CAPACITANCE AND RESISTANCE IN SERIES

3.1 L-C-R series circuit

A circuit containing a series combination of an resistance R, a coil of inductance L and a capacitor of capacitance C, connected with a source of alternating e.m.f. of peak value of E_0, as shown in figure.



- **Phasor Diagram For Series L-C-R circuit**

Let in series LCR circuit applied alternating emf is $E = E_0 \sin \omega t$.As L,C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase.

However voltage across each element bears a different phase relationship with the current.

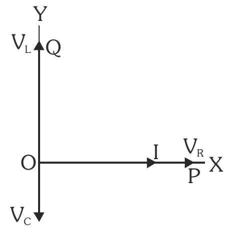
Let at any instant of time t the current in the circuit is I.

Let at this time t the potential differences across L, C, and R

$V_L = I X_L$, $V_C = I X_C$ and $V_R = IR$

Now, V_R is in phase with current I but V_L leads I by 90°

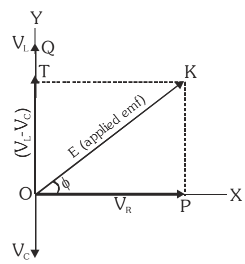
While V_C lags behind I by 90°.



The vector OP represents V_R (which is in phase with I) the vector OQ represent V_L (which leads I by 90°) and the vector OS represents V_C (which legs behind I by 90°)

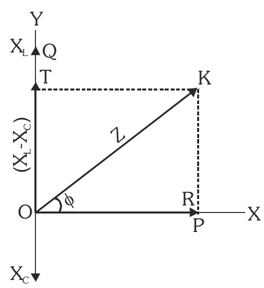
V_L and V_C are opposite to each other.

If $V_L > V_C$ (as shown in figure) the their resultant will be $(V_L - V_C)$ which is represented by OT. Finally, the vector OK represents the resultant of V_R and $(V_L - V_C)$, that is, the resultant of all the three = applied e.m.f.



Thus $E = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2} \Rightarrow I = \dfrac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

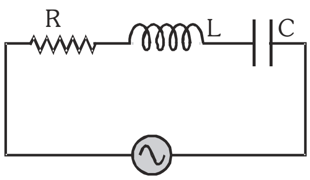


The phasor diagram also shown that in LCR circuit the applied e.m.f. leads the current I by a phase angle ϕ $\quad \tan\phi = \dfrac{X_L - X_C}{R}$

3.2 Series LCR and parallel LCR combination

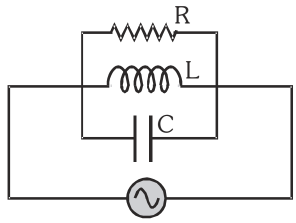
<table>
<tr><th>SERIES L-C-R CIRCUIT</th><th>PARALLEL L-C-R CIRCUIT</th></tr>
<tr><td>

1. Circuit diagram



I same for R, L & C

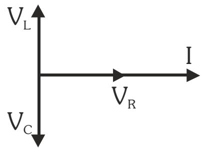
</td><td>

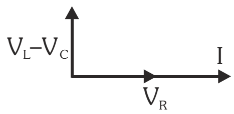


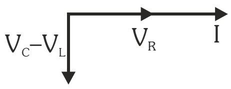
V same for R, L and C

</td></tr>
<tr><td>

2. Phasor diagram



(i) If $V_L > V_C$ then 

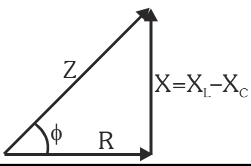
(ii) If $V_C > V_L$ then 

(iii) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{V_L - V_C}{V_R}$

(iv) Impedance triangle



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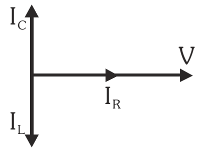
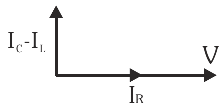
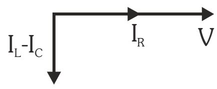


(i) if $I_C > I_L$ then 

(ii) if $I_L > I_C$ then 

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</table>

3.3 Resonance

A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

There are two types of resonance : (i) Series Resonance (ii) Parallel Resonance

3.4 Series Resonance

(a) At Resonance

(i) $X_L = X_C$ (ii) $V_L = V_C$ (iii) $\phi = 0$ (V and I in same phase)

(iv) $Z_{min} = R$ (impedance minimum) (v) $I_{max} = \dfrac{V}{R}$ (current maximum)

(b) Resonance frequency

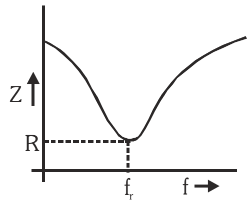
$$\because\ X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 = \frac{1}{LC} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

(c) Variation of Z with f

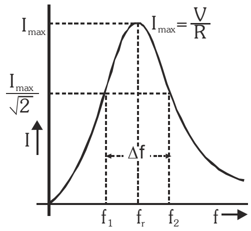
(i) If $f < f_r$ then $X_L < X_C$ circuit nature capacitive, ϕ (negative)

(ii) At $f = f_r$ then $X_L = X_C$ circuit nature, Resistive, ϕ = zero

(iii) If $f > f_r$ then $X_L > X_C$ circuit nature is inductive, ϕ (positive)



(d) Variation of I with f as f increase, Z first decreases then increase



as f increase, I first increase then decreases

- At resonance, impedance of the series resonant circuit is minimum so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.
- **Half power frequencies**

  The frequencies at which, power become half of its maximum value is called half power frequencies
- **Band width** $= \Delta f = f_2 - f_1$
- **Quality factor Q** : Q-factor of AC circuit basically gives an idea about stored energy & lost energy.

$$Q = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{maximum energy loss per cycle}}$$

(i) It represents the sharpness of resonance. (ii) It is unit less and dimension less quantity

(iii) $Q = \frac{(X_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{2\pi f_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{f_r}{\Delta f} = \frac{f_r}{\text{band width}}$

Magnification

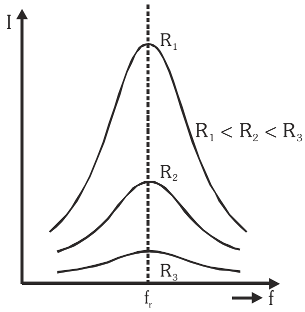
At resonance V_L or V_C = QE (where E = supplied voltage)

So at resonance Magnification factor = Q-factor

Sharpness

Sharpness $\propto$ Quality factor $\propto$ Magnification factor

R decrease $\Rightarrow$ Q increases $\Rightarrow$ Sharpness increases

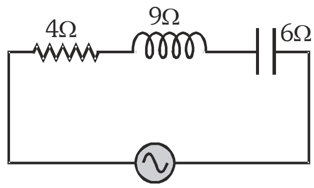


GOLDEN KEY POINTS

- In A.C. circuit voltage for L or C may be greater than source voltage or current but it happens only when circuit contains L and C both and in R it is never greater than source voltage or current.
- In parallel A.C.circuit phase difference between I_L and I_C is π
- Series resonance circuit gives voltage amplification while parallel resonance circuit gives current amplification.
- At resonance current does not depend on L and C, it depends only on R and V.
- At half power frequencies : net reactance = net resistance.
- As R increases, bandwidth increases
- To obtain resonance in a circuit following parameter can be altered :

  (i) L (ii) C (iii) frequency of source.
- Two series LCR circuit of same resonance frequency f are joined in series then resonance frequency of series combination is also f
- The series resonance circuit called acceptor whereas parallel resonance circuit called rejector circuit.
- Unit of $\sqrt{LC}$ is second

Illustration 20.



Find out the impedance of given circuit.

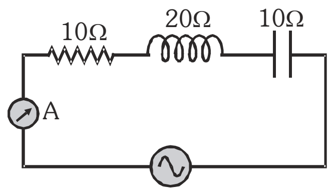
Solution

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{4^2+(9-6)^2}=\sqrt{4^2+3^2}=\sqrt{25}=5\Omega \qquad (\because X_L > X_C \therefore \text{Inductive})$$

Illustration 21.

Find out reading of A C ammeter and also calculate the potential difference across, resistance and capacitor.

Solution



E=100sinωt100πt volt

$$Z=\sqrt{R^2+(X_L-X_C)^2}=10\sqrt{2}\ \Omega \qquad \Rightarrow \qquad I_0=\frac{V_0}{Z}=\frac{100}{10\sqrt{2}}=\frac{10}{\sqrt{2}}A$$

$\because$ ammeter reads RMS value, so its reading $=\dfrac{10}{\sqrt{2}\,\sqrt{2}}=5A$

so $V_R = 5 \times 10 = 50$ V and $V_C = 5 \times 10 = 50$ V

Illustration 22.

In LCR circuit with an AC source R = 300 Ω , C = 20 μF, L = 1.0 H, E_{rms} = 50V and f = 50/π Hz. Find RMS current in the circuit.

Solution

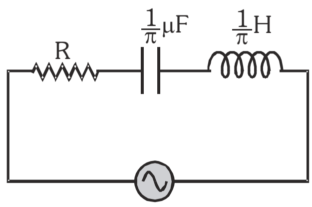
$$I_{rms}=\frac{E_{rms}}{Z}=\frac{E_{rms}}{\sqrt{R^2+\left[\omega L-\frac{1}{\omega C}\right]^2}}=\frac{50}{\sqrt{300^2+\left[2\pi\times\frac{50}{\pi}\times 1-\frac{1}{20\times10^{-6}\times2\pi\times\frac{50}{\pi}}\right]^2}}$$

$$\Rightarrow \quad I_{rms}=\frac{50}{\sqrt{(300)^2+\left[100-\frac{10^3}{2}\right]^2}}=\frac{50}{100\sqrt{9+16}}=\frac{1}{10}=0.1A$$

Illustration 23.

For what frequency the voltage across the resistance R will be maximum.

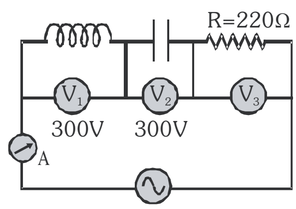
Solution



It happens at resonance

$$f=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{\frac{1}{\pi}\times10^{-6}\times\frac{1}{\pi}}}=500\ \text{Hz}$$

Illustration 24.



110V, 60Hz

A capacitor, a resistor and a 40 mH inductor are connected in series to an AC source of frequency 60Hz, calculate the capacitance of the capacitor, if the current is in phase with the voltage. Also find the reading of voltmeter V_3 and Ammeter.

Solution

At resonance $\omega L=\dfrac{1}{\omega C}$, $C=\dfrac{1}{\omega^2 L}=\dfrac{1}{4\pi^2 f^2 L}=\dfrac{1}{4\pi^2\times(60)^2\times40\times10^{-3}}=176\mu F$

$$V_3 = V_R \Rightarrow V_3 = 110\ \text{V} \quad \text{and} \quad I=\frac{V}{R}=\frac{110}{220}=0.5A$$

Illustration 25.

A coil, a capacitor and an A.C. source of rms voltage 24 V are connected in series, By varying the frequency of the source, a maximum rms current 6 A is observed, If this coil is connected to a bettery of emf 12 V, and internal resistance 4Ω , then calculate the current through the coil.

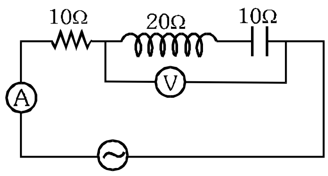
Solution

At resonance current is maximum. $I = \frac{V}{R} \Rightarrow$ Resistance of coil $R = \frac{V}{I} = \frac{24}{6} = 4\ \Omega$

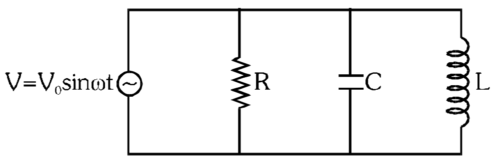
When coil is connected to battery, suppose I current flow through it then $I = \frac{E}{R+r} = \frac{12}{4+4} = 1.5$ A

BEGINNER'S BOX-3

1. In given circuit find the reading of ammeter and voltmeter.



2. For the circuit shown in figure, write down the instantaneous current through each element.

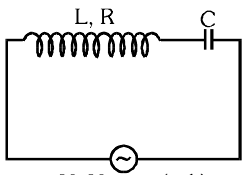


3. A variable frequency 230V alternating voltage source is connected across a series combination of L = 5.0H C = 80μF and R = 40Ω. Calculate

(a) The angular frequency of the source at resonance. (b) Amplitude of current at resonance frequency

4. A coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c. voltage, Circuit resonates when angular frequency of supply is $\omega = \omega_0$.

Find out relation betwen ω_0, L and C



5. Find the phase difference between voltage and current in series LCR circuit at half power frequencies.

6. A series LCR circuit with L = 0.12H, C = 480 nF, R = 23Ω is connected to a 230 V variable frequency supply Find –

(a) Source frequency for which current is maximum. (b) Q–factor of the given circuit.

4. POWER IN AC CIRCUIT

4.1 The average power dissipation in LCR AC circuit

Let $V = V_0 \sin\omega t$ and $I = I_0 \sin(\omega t - \phi)$

Instantaneous power $P = (V_0 \sin\omega t)(I_0 \sin(\omega t - \phi)) = V_0 I_0 \sin\omega t (\sin\omega t\cos\phi - \sin\phi\cos\omega t)$

$$\text{Average power } \langle P\rangle = \frac{1}{T}\int_0^T (V_0 I_0 \sin^2\omega t\cos\phi - V_0 I_0 \sin\omega t\cos\omega t\sin\phi)dt$$

$$= V_0 I_0\left[\frac{1}{T}\int_0^T \sin^2\omega t\cos\phi dt - \frac{1}{T}\int_0^T \sin\omega t\cos\omega t\sin\phi dt\right] = V_0 I_0\left[\frac{1}{2}\cos\phi - 0\times\sin\phi\right]$$

$$\Rightarrow \quad \langle P\rangle = \frac{V_0 I_0\cos\phi}{2} = V_{rms}\, I_{rm,s}\, \cos\phi$$

Instantaneous power	Average power/actual power/ dissipated power/power loss	Virtual power/ apparent Power/rms Power	Peak power
$P = VI$	$P = V_{rms}\, I_{rms} \cos\phi$	$P = V_{rms}\, I_{rms}$	$P = V_0\, I_0$

- $I_{rms}\cos\phi$ is known as active part of current or wattfull current, workfull current. It is in phase with voltage.
- $I_{rms}\sin\phi$ is known as inactive part of current, wattless current, workless current. It is in quadrature (90°) with voltage.

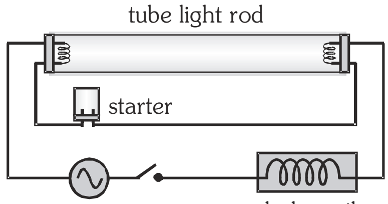
4.2 Power factor :

Average power $\bar{P} = E_{rms}\, I_{rms}\cos\phi = \text{rms power} \times \cos\phi$

Power factor $(\cos\phi) = \dfrac{\text{Average power}}{\text{rms Power}}$ and $\cos\phi = \dfrac{R}{Z}$

4.3 Choke Coil

In a direct current circuit, current is reduced with the help of a resistance. Hence there is a loss of electrical energy $I^2 R$ per sec in the form of heat in the resistance. But in an AC circuit the current can be reduced by choke coil which involves very small amount of loss of energy. Choke coil is a copper coil wound over a soft iron laminated core. This coil is put in series with the circuit in which current is to be reduced.



Circuit with a choke coil is a series L-R circuit. If resistance of choke coil = r (very small)

The current in the circuit $I = \dfrac{E}{Z}$ with $Z = \sqrt{(R+r)^2 + (\omega L)^2}$ So due to large inductance L of the coil, the current in the circuit is decreased appreciably. However, due to small resistance of the coil r,

The power loss in the choke $P_{av} = V_{rms}\, I_{rms}\cos\phi \to 0$

$$\because \quad \cos\phi = \frac{r}{Z} = \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \frac{r}{\omega L} \to 0$$

GOLDEN KEY POINTS

- $P_{av} \le P_{rms}$.
- Power factor varies from 0 to 1
-

Pure/Ideal	V	Power factor = $\cos\phi$	Average power
R	V, I same Phase	1 (maximum)	$V_{rms} \cdot I_{rms}$
L	V leads I by $\frac{\pi}{2}$	0, lagging	0
C	V lags I by $\frac{\pi}{2}$	0, leading	0
Choke coil	V leads I by $\frac{\pi}{2}$	0, lagging	0

- At resonance power factor is maximum ($\phi = 0$ so $\cos\phi = 1$) and $P_{av} = V_{rms}\, I_{rms}$
- Choke coil is an inductor having high inductance and negligible resistance.
- Choke coil is used to control current in A.C. circuit at negligible power loss
- Choke coil used only in A.C. and not in D.C. circuit
- Choke coil is based on the principle of wattless current.
- Iron cored choke coil is used generally at low frequency and air cored at high frequency.
- Resistance of ideal choke coil is zero

Illustrations

Illustration 26.

A voltage of 10 V and frequency 10^3 Hz is applied to $\frac{1}{\pi}$ μF capacitor in series with a resistor of 500Ω. Find the power factor of the circuit and the power dissipated.

Solution

$$\because \quad X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10^3 \times \frac{10^{-6}}{\pi}} = 500\Omega \quad \therefore Z = \sqrt{R^2 + X_C^2} = \sqrt{(500)^2 + (500)^2} = 500\sqrt{2}\,\Omega$$

$$\text{Power factor } \cos\phi = \frac{R}{Z} = \frac{500}{500\sqrt{2}} = \frac{1}{\sqrt{2}}, \quad \text{Power dissipated} = V_{rms}\, I_{rms} \cos\phi = \frac{V_{rms}^2}{Z}\cos\phi = \frac{(10)^2}{500\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{10}\text{ W}$$

Illustration 27.

If V = 100 sin 100 t volt and I = 100 sin (100 t + $\frac{\pi}{3}$) mA for an A.C. circuit then find out

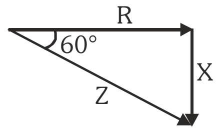
(a) phase difference between V and I
(b) total impedance, reactance, resistance
(c) power factor and power dissipated
(d) components contains by circuits

Solution

(a) Phase difference $\phi = -\frac{\pi}{3}$ (I leads V)

(b) Total impedance $Z = \frac{V_0}{I_0} = \frac{100}{100 \times 10^{-3}} = 1k\Omega$ Now resistance $R = Z\cos 60^\circ = 1000 \times \frac{1}{2} = 500\Omega$

reactance $X = Z\sin 60^\circ = 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3}\Omega$



(c) $\phi = -60^\circ \Rightarrow$ Power factor $= \cos\phi = \cos(-60^\circ) = 0.5$ (leading)

Power dissipated $P = V_{rms} I_{rms} \cos\phi = \frac{100}{\sqrt{2}} \times \frac{0.1}{\sqrt{2}} \times \frac{1}{2} = 2.5\,W$

(d) Circuit must contains R as $\phi \neq \frac{\pi}{2}$ and as ϕ is negative so C must be there, (L may exist but $X_C > X_L$)

Illustration 28.

If power factor of a R-L series circuit is $\frac{1}{2}$ when applied voltage is V = 100 sin 100πt volt and resistance of circuit is 200Ω then calculate the inductance of the circuit.

Solution

$$\cos\phi = \frac{R}{Z} \Rightarrow \frac{1}{2} = \frac{R}{Z} \Rightarrow Z = 2R \Rightarrow \sqrt{R^2 + X_L^2} = 2R$$

$$\Rightarrow X_L = \sqrt{3}\,R$$

$$\omega L = \sqrt{3}\,R \Rightarrow L = \frac{\sqrt{3}R}{\omega} = \frac{\sqrt{3} \times 200}{100\pi} = \frac{2\sqrt{3}}{\pi}H$$

Illustration 29.

A circuit consisting of an inductance and a resistacne joined to a 200 volt supply (A.C.). It draws a current of 10 ampere. If the power used in the circuit is 1500 watt. Calculate the wattless current.

Solution

Apparent power = 200 × 10 = 2000 W

$\therefore$ Power factor $\cos\phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{1500}{2000} = \frac{3}{4}$

Wattless current $= I_{rms} \sin\phi = 10\sqrt{1 - \left(\frac{3}{4}\right)^2} = \frac{10\sqrt{7}}{4}A$

Illustration 30.

A choke coil and a resistance are connected in series in an a.c circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance is 50 V. What would be the potential difference across the choke coil.

Solution

$$V = \sqrt{V_R^2 + V_L^2} \Rightarrow V_L = \sqrt{V^2 - V_R^2} = \sqrt{(130)^2 - (50)^2} = 120\text{ V}$$

Illustration 31.

An electric lamp which runs at 80V DC consumes 10 A current. The lamp is connected to 100 V – 50 Hz ac source compute the inductance of the choke required.

Solution

Resistance of lamp $R = \frac{V}{I} = \frac{80}{10} = 8\Omega$

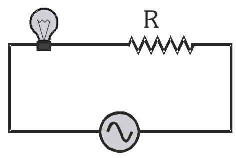
Let Z be the impedance which would maintain a current of 10 A through the Lamp when it is run on 100 Volt a.c. then.$Z = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$ but $Z = \sqrt{R^2 + (\omega L)^2}$

$\Rightarrow \quad (\omega L)^2 = Z^2 - R^2 = (10)^2 - (8)^2 = 36 \Rightarrow \omega L = 6 \qquad \Rightarrow L = \frac{6}{\omega} = \frac{6}{2\pi \times 50} = 0.02H$

Illustration 32.

Calculate the resistance or inductance required to operate a lamp (60V, 10W) from a source of (100 V, 50 Hz)

Solution

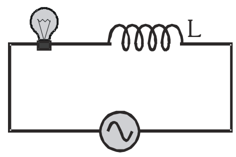


(a) Maximum voltage across lamp = 60V

$\because \quad V_{Lamp} + V_R = 100 \qquad \therefore \quad V_R = 40V$

Now current througth Lamp is $= \frac{\text{Wattage}}{\text{voltage}} = \frac{10}{60} = \frac{1}{6}A$

But $V_R = IR \quad \Rightarrow \quad 40 = \frac{1}{6}(R) \Rightarrow \quad R = 240\ \Omega$



(b) Now in this case $(V_{Lamp})^2 + (V_L)^2 = (V)^2$

$(60)^2 + (V_L)^2 = (100)^2 \Rightarrow V_L = 80\ V$

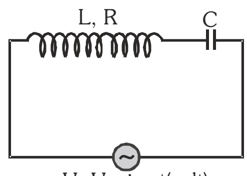
Also $V_L = IX_L = \frac{1}{6}X_L$ so $X_L = 80 \times 6 = 480\ \Omega = L\ (2\pi f) \Rightarrow L = 1.5\ H$

A capacitor of suitable capacitance replace a choke coil in an AC circuit, the average power consumed in a capacitor is also zero. Hence, like a choke coil, a capacitor can reduce current in AC circuit without power dissipation.

Cost of capacitor is much more than the cost of inductance of same reactance that's why choke coil is used.

Illustration 33.

A choke coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c. voltage, Circuit resonates when angular frequency of supply is $\omega = \omega_0$.



(a) Find out relation betwen ω_0, L and C

(b) What is phase difference between V and I at resonance, is it changes when resistance of choke coil is zero.

Solution

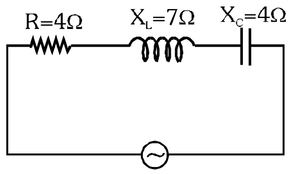
(a) At resonance condition $X_L = X_C \Rightarrow \omega_0 L = \dfrac{1}{\omega_0 C} \Rightarrow \omega_0 = \dfrac{1}{\sqrt{LC}}$

(b) $\because \cos\phi = \dfrac{R}{Z} = \dfrac{R}{R} = 1 \quad \therefore \phi = 0°$ No, It is always zero.

BEGINNER'S BOX-4

1. What is the power factor of a circuit that draws 5A at 160 V and whose power consumption is 600W?

2. In a series LCR circuit as shown in fig.



E = 25 sin (100πt) volt

(a) Find heat developed in 80 seconds

(b) Find wattless current

3. For a series LCR circuit

$I = 100\sin(100\pi t - \pi/3)$mA and $V = 100\sin(100\pi t)$volt, then

(a) Calculate resistance and reactance of circuit.

(b) Find average power loss.

4. The source voltage and current in the circuit are represented by the following equations –

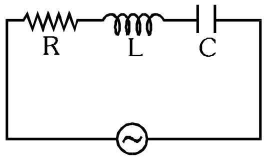
$E = 110 \sin\left(\omega t + \dfrac{\pi}{6}\right)$ volt, $I = 5\sin\left(\omega t - \dfrac{\pi}{6}\right)$ ampere

Find :–

(a) Impedance of circuit.

(b) Power factor with nature

5. In given circuit R = 100Ω. If voltage leads current by 60° then find –



V=200sin100πt volt

(a) Current supply by source.

(b) Average power

6. An inductor of reactance 4Ω and a resistor of resistance 3Ω are connected in series with 100V ac supply, calculate wattless current in circuit.

7. A 100 Ω resistor is connected to a 220 V, 50 Hz a.c. supply.

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

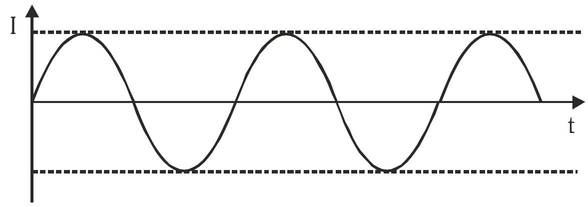
8. A choke coil and a resistance are connected in series in an a.c. circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance is 50V. What would be the potential difference across the choke coil.

5. LC OSCILLATION

The oscillation of energy between capacitor (electric field energy) and inductor (magnetic field energy) is called LC Oscillation.

5.1 Undamped oscillation

When the circuit has no resistance, the energy taken once from the source and given to capacitor keeps on oscillating between C and L then the oscillation produced will be of constant amplitude. These are called undamped oscillation.



After switch is closed

$$\frac{Q}{C}+L\frac{di}{dt}=0 \Rightarrow \frac{Q}{C}+L\frac{d^2Q}{dt^2}=0 \Rightarrow \frac{d^2Q}{dt^2}+\frac{1}{LC}Q=0$$

By comparing with standard equation of free oscillation $\left[\frac{d^2x}{dt^2}+\omega^2x=0\right]$

$\omega^2=\frac{1}{LC}$ Frequency of oscillation $f=\frac{1}{2\pi\sqrt{LC}}$

Charge varies sinusoidally with time $q = q_m \cos \omega t$

current also varies periodically with t $I = \frac{dq}{dt} = q_m \omega \cos\left(\omega t+\frac{\pi}{2}\right)$

If initial charge on capacitor is q_m then electrical energy strored in capacitor is $U_E = \frac{1}{2}\frac{q_m^2}{C}$

At t = 0 switch is closed, capacitor starts to discharge.

As the capacitor is fully discharged, the total electrical energy is stored in the inductor in the form of magnetic energy.

$$U_B = \frac{1}{2}LI_m^2 \qquad \text{where } I_m = \text{max. current}$$

$$(U_{max})_{EPE} = (U_{max})_{MPE} \qquad \Rightarrow \frac{1}{2}\frac{q_m^2}{C}=\frac{1}{2}LI_m^2$$

GOLDEN KEY POINTS

- In damped oscillation amplitude of oscillation decreases exponentially with time.
- At $t=\frac{T}{4},\frac{3T}{4},\frac{5T}{4}$..... energy stored is completely magnetic.
- At $t=\frac{T}{8},\frac{3T}{8},\frac{5T}{8}$..... energy is shared equally between L and C
- Phase difference between charge and current is $\frac{\pi}{2}$ $\left[\begin{array}{l}\text{when charge is maximum, current minimum}\\ \text{when charge is minimum, current maximum}\end{array}\right]$

Illustration 34.

An LC circuit contains a 20mH inductor and a 50μF capacitor with an initial charge of 10mC. The resistance of the circuit is negligible. Let the instant the circuit is closed to be t = 0.

(a) What is the total energy stored initially.

(b) What is the natural frequency of the circuit.

(c) At what minimum time is the energy stored is completely magnetic.

(d) At what minimum time is the total energy shared equally between inductor and the capacitor.

Solution

(a) $U_E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} \times \frac{(10 \times 10^{-3})^2}{50 \times 10^{-6}} = 1.0\text{J}$

(b) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} = 10^3 \text{ rad/sec} \quad \Rightarrow \quad f = 159 \text{ Hz}$

(c) $\because \quad q = q_0 \cos \omega t$

Energy stored is completely magnetic (i.e. electrical energy is zero, q = 0)

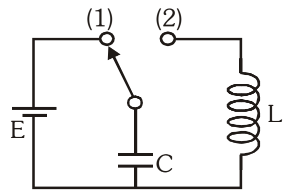
at $t = \frac{T}{4}$, where $T = \frac{1}{f} = 6.3 \text{ ms}$

(d) Energy is shared equally between L and C when charge on capacitor become $\frac{q_0}{\sqrt{2}}$

so, at $t = \frac{T}{8}$, energy is shared equally between L and C

BEGINNER'S BOX-5

1. Initially key was placed on (1) till the capacitor got fully charged. Key is placed on (2) at t=0. The minimum time when the energy in both capacitor and inductor will be same-



2. An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting L–C circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of Q is 200 μC. **[IIT-JEE 2006]**

(i) Find the maximum value of I.

(ii) When Q = 200 μC, what is the value of I?

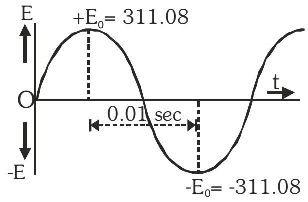
(iii) When Q = 100 μC, what is the value of | dI/dt | ?

(iv) When I is equal to one–half its maximum value, what is the value of | Q | ?

ANSWERS

BEGINNER'S BOX-1

1. There are two reasons for it :



2. For AC, $I = I_0 \sin\omega t$, the instantaneous value of heat produced (per second) in a resistance R is,

$H = I^2R = I_0^2\sin^2\omega t \times R$ the average value of heat produced during a cycle is :

$$H_{av} = \frac{\int_0^T H\,dt}{\int_0^T dt} = \frac{\int_0^T (I_0^2\sin^2\omega t \times R)dt}{\int_0^T dt} = \frac{1}{2}I_0^2R$$

$$\left[\because \int_0^T I_0^2 \sin^2\omega t\,dt = \frac{1}{2}I_0^2T\right]$$

$$\Rightarrow H_{av} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = I_{rms}^2 R \;.....(i)$$

However, in case of DC, $H_{DC} = I^2 R$...(ii)

$\because I = I_{rms}$ so from equation (i) and (ii) $H_{DC} = H_{av}$

AC produces same heating effects as DC of value $I = I_{rms}$. This is also why AC instruments which are based on heating effect of current give rms value.

3. The average value of a.c. for a cycle is zero. So a d.c. ammeter will always read zero in a.c. circuit.

4. 2.5 m s **5.** (a) 0.01 s, (b) 60°

6. 200 V

BEGINNER'S BOX-2

1. 2.1 A **2.** $10000\sin\left(1000t - \frac{\pi}{2}\right)$ A

3. 1A

4. The capacitive reactance $X_C = 212\ \Omega$

$I_{rms} = 1.03$ A

The peak current $I_0 = 1.46$ A

If the frequency is doubled the capacitive reactances is halved, the consequently, the current is doubled.

5. 2.49 A **6.** (a) resistor (b) inductor

7. $R = 5\Omega$ and $X_L = 5\sqrt{3}\ \Omega$

8. 480Ω **9.** 1H **10.** 0.08 henry

11. 240.2 Ω **12.** (a) 2.4 A (b) 2.5 ms

13. 15.9 mH

14. (A) – r, (B) – p, (C) – p, (D) – p, (E) – r

BEGINNER'S BOX-3

1. Reading of ammeter = 2·5A

Reading of voltmeter = 25V

2. The three current equations are,

$$V = i_R R,\quad V = L\frac{di_L}{dt} \text{ and } \frac{dV}{dt} = \frac{1}{C}i_C$$

so $i_R = \frac{V_0}{R}\sin\omega t$,

$i_L = -\frac{V_0}{\omega L}\cos\omega t$ and $i_C = V_0\omega C \cos\omega t$

3. (a) angular frequancy at resonance $\omega_r = 50$ rad/s

(b) amplitude of current at resonance $I_m = 8.13$ A

4. (a) $\omega_0 = \frac{1}{\sqrt{LC}}$, (b) $\phi = 0°$ No, It is always zero.

5. $\phi = \frac{\pi}{4}$

6. (a) 6.63×10^2 Hz (b) Quality factor Q = 21.7

BEGINNER'S BOX-4

1. 0.75

2. (a) 4000 joule (b) 2.12 A

3. (a) R = 500 ohm, X = $500\sqrt{3}$ ohm (b) 2.5 watts

4. (a) Impedance Z = 22Ω (b) Power factor = $\frac{1}{2}$ (lagging)

5. (a) $\frac{1}{\sqrt{2}}$ A, (b) 50W

6. Wattless current = 16A

7. (a) 2.2 A (b) 484 watt

8. 120V

BEGINNER'S BOX-5

1. $t = \frac{\pi\sqrt{LC}}{4}$

2. (i) 2.0 A, (ii) Zero, (iii) 10^4 A/s, (iv) 1.732×10^{-4} C

EXERCISE-I (Conceptual Questions)

PEAK, AVERAGE AND RMS VALUE

1. What is the r.m.s. value of an alternating current which when passed through a resistor produces heat which is thrice of that produced by a direct current of 2 amperes in the same resistor :-

(1) 6 amp (2) 2 amp

(3) 3.46 amp (4) 0.66 amp

2. The peak value of an alternating e.m.f. which is given by $E = E_0 \cos\omega t$ is 10 volts and its frequency is 50 Hz. At time $t = \frac{1}{600}$ s, the instantaneous e.m.f. is

(1) 10 V (2) $5\sqrt{3}$ V (3) 5 V (4) 1V

3. The phase difference between current and voltage in an AC circuit is $\frac{\pi}{4}$ radian, If the frequency of AC is 50 Hz, then the phase difference is equivalent to the time difference:-

(1) 0.78 s (2) 15.7 ms (3) 2.5 s (4) 2.5 ms

4. A current in circuit is given by $i = 3 + 4 \sin \omega t$. Then the effective value of current is :

(1) 5 (2) $\sqrt{7}$ (3) $\sqrt{17}$ (4) $\sqrt{10}$

5. Incorrect statement are :

(a) A.C. meters can measure D.C also

(b) If A.C. meter measures D.C. there scale must be linear and uniform

(c) A.C. and D.C. meters are based on heating effect of current

(d) A.C. meter reads rms value of current

(1) a,b (2) b,c (3) c,d (4) d,a

6. The r.m.s. value of current for a variable current $i=i_1 \cos \omega t + i_2 \sin \omega t$:–

(1) $\frac{1}{\sqrt{2}}(i_1 + i_2)$ (2) $\frac{1}{\sqrt{2}}(i_1 + i_2)^2$

(3) $\frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$ (4) $\frac{1}{2}(i_1^2 + i_2^2)^{1/2}$

7. The relation between an A.C. voltage source and time in SI units is :

$V = 120 \sin (100\,\pi t) \cos (100\,\pi t)$ volt value of peak voltage and frequency will be respectively :–

(1) 120 volt and 100 Hz

(2) $\frac{120}{\sqrt{2}}$ volt and 100 Hz

(3) 60 volt and 200 Hz

(4) 60 volt and 100 Hz

8. If an A.C. main supply is given to be 220 V. What would be the average e.m.f. during a positive half cycle :-

(1) 198 V (2) 386 V

(3) 256 V (4) None of these

9. The hot wire ammeter measures :-

(1) D.C. current (2) A.C. current

(3) None of above (4) both (1) & (2)

10. Frequency of A.C. in India is –

(1) 45 Hz (2) 60 Hz

(3) 50 Hz (4) None of the above

11. For an alternating current $I = I_0 \cos \omega t$, What is the rms value and peak value of current :-

(1) $I_0, \frac{I_0}{\sqrt{2}}$ (2) $\frac{I_0}{\sqrt{2}}, I_0$

(3) $I_0, \frac{I_0}{2}$ (4) $2I_0, \frac{I_0}{\sqrt{2}}$

12. If a step up transformer have turn ratio 5, frequency 50 Hz root mean square value of potential difference on primary 100 volts and the resistance of the secondary winding is 500 Ω then the peak value of voltage in secondary winding will be (the efficiency of the transformer is hundred percent)

(1) $500\sqrt{2}$ (2) $10\sqrt{2}$

(3) $50\sqrt{2}$ (4) $20\sqrt{2}$

SIMPLE AC CIRCUIT

13. A resonant A.C. circuit contains a capacitor of capacitance 10^{-6} *F* and an inductor of 10^{-4} *H*. The frequency of electrical oscillations will be :-

(1) 10^5 Hz (2) 10 Hz

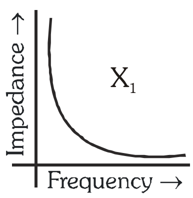
(3) $\frac{10^5}{2\pi}$ Hz (4) $\frac{10}{2\pi}$ Hz

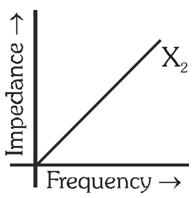
14. A resistance of 300Ω and an inductance of $\frac{1}{\pi}$ henry are connected in series to a A.C. voltage of 20 volts and 200 Hz frequency. The phase angle between the voltage and current is :-

(1) $\tan^{-1}\left(\frac{4}{3}\right)$ (2) $\tan^{-1}\left(\frac{3}{4}\right)$

(3) $\tan^{-1}\left(\frac{3}{2}\right)$ (4) $\tan^{-1}\left(\frac{2}{3}\right)$

15. The graphs given below depict the dependence of two reactive impedances X_1 and X_2 on the frequency of the alternating e.m.f. applied individually to them. We can then say that :





(1) X_1 is an inductor and X_2 is a capacitor
(2) X_1 is a resistor and X_2 is a capacitor
(3) X_1 is a capacitor and X_2 is an inductor
(4) X_1 is an inductor and X_2 is a resistor

16. A 12 ohm resistor and a 0.21 henry inductor are connected in series to an AC source operating at 20 volts, 50 cycle/second. The phase angle between the current and the source voltage is:

(1) 30° (2) 40° (3) 80° (4) 90°

17. A 110 V, 60 W lamp is run from a 220 V AC mains using a capacitor in series with the lamp, instead of a resistor then the voltage across the capacitor is about:-

(1) 110 V (2) 190 V (3) 220 V (4) 311 V

18. The resistance that must be connected in series with inductance of 0.2 H in order that the phase difference between current and e.m.f. may be 45° when the frequency is 50 Hz, is:-

(1) 6.28 ohm. (2) 62.8 ohm.
(3) 628 ohm. (4) 31.4 ohm.

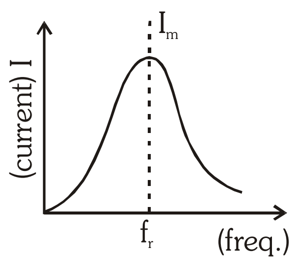
19. An inductive circuit contains resistance of 10 ohms and an inductance of 20 H. If an A.C. voltage of 120 volt and frequency 60 Hz is applied to this circuit, the current would be nearly

(1) 0.016 amp. (2) 0.16 amp.
(3) 0.48 amp. (4) 0.80 amp.

20. A student connects a long air cored - coil of manganin wire to a 100 V D.C. supply and records a current of 25 amp. When the same coil is connected across 100 V. 50 Hz a.c. the current reduces to 20 A , the reactance of the coil is :-

(1) 4 Ω (2) 3 Ω (3) 5 Ω (4) None

21. The graph shows variation of I with f for a series R-L-C network. Keeping L and C constant. If R decreases :



(a) Maximum current (I_m) increases
(b) Sharpness of the graph increases
(c) Quality factor increases
(d) Band width increases

(1) a, b, c (2) b, c, d (3) c, d, a (4) All

22. Alternating current is flowing in inductance L and resistance R. The frequency of source is ω/2π. Which of the following statement is correct :

(1) For low frequency the limiting value of impedance is L.
(2) For high frequency the limiting value of impedance is ωL.
(3) For high frequency the limiting value of impedance is R.
(4) For low frequency the limiting value of impedance is ωL.

23. A bulb and a capacitor are connected in series to a source of alternating current. If its frequency is increased, while keeping the voltage of the source constant, then

(1) Bulb will give more intense light.

(2) Bulb will give less intense light.

(3) Bulb will give light of same intensity as before

(4) Bulb will stop radiating light.

24. In an A.C. circuit resistance and inductance are connected in series. The potential and current in inductance is:

(1) $V_0 \sin \omega t, \dfrac{V_0}{\omega L} \sin\omega t$

(2) $V_0 \sin \omega t, \dfrac{V_0}{\omega L} \sin(\omega t + \pi/2)$

(3) $V_0 \sin(\omega t + \pi/2), \dfrac{V_0}{\omega L} \sin\omega t$

(4) $V_0 \sin(\omega t + \pi/2), \dfrac{V_0}{\omega L} \sin(\omega t - \pi/2)$

25. An a.c. source of voltage V and of frequency 50 Hz is connected to an inductor of 2 H and negligible resistance. A current of r.m.s value I flows in the coil. When the frequency of the voltage is changed to 400 Hz keeping the magnitude of V the same, the current is now :-

(1) 8 I in phase with V

(2) 4 I and leading by 90° from V

(3) $\dfrac{I}{4}$ and lagging by 90° from V

(4) $\dfrac{I}{8}$ and lagging by 90° from V

26. A capacitor of capacity C is connected in A.C. circuit. The applied emf is $V = V_0 \sin\omega t$, then the current is :

(1) $I = \dfrac{V_0}{\omega L} \sin\omega t$

(2) $I = \dfrac{V_0}{\omega L} \sin(\omega t + \pi/2)$

(3) $I = V_0 \omega C \sin\omega t$

(4) $I = V_0 \omega C \sin(\omega t + \pi/2)$

27. The impedence of a circuit, when a resistance R and an inductor of inductance L are connected in series in an A.C. circuit of frequency (f) is :-

(1) $\sqrt{R + 4\pi f L^2}$ (2) $\sqrt{R + 4\pi^2 f^2 L^2}$

(3) $\sqrt{R^2 + 4\pi^2 f^2 L^2}$ (4) $\sqrt{R^2 + 2\pi^2 f^2 L^2}$

28. A capacitor of capacity C and reactance X if capacitance and frequency become double then reactance will be :–

(1) 4X (2) $\dfrac{X}{2}$ (3) $\dfrac{X}{4}$ (4) 2X

29. The coil of choke in a circuit :

(1) increases the current

(2) controled the current

(3) has high resistance to d.c. circuit

(4) does not change the current

30. The inductive reactance of an inductive coil with $\dfrac{1}{\pi}$ henry and 50 Hz :–

(1) $\dfrac{50}{\pi}$ ohm (2) $\dfrac{\pi}{50}$ ohm

(3) 100 ohm (4) 50 ohm

31. In the L–R circuit R = 10Ω and L = 2H. If 120 V, 60 Hz alternating voltage is applied then the flowing current in this circuit will be :-

(1) 0.32 A (2) 0.16 A (3) 0.48 A (4) 0.80 A

32. An inductance of 0.4 Henry and a resistance of 100 ohm are connected to a A.C. voltage source of 220 V and 50 Hz. Then find out the phase difference between the voltage and current flowing in the circuit :

(1) $\tan^{-1}(2.25\,\pi)$ (2) $\tan^{-1}(0.4\,\pi)$

(3) $\tan^{-1}(1.5\,\pi)$ (4) $\tan^{-1}(0.5\,\pi)$

33. A capacitor of capacitance 100 μF & a resistance of 100Ω is connected in series with AC supply of 220V, 50Hz. The current leads the voltage by ....

(1) $\tan^{-1}\left(\frac{1}{2\pi}\right)$ (2) $\tan^{-1}\left(\frac{1}{\pi}\right)$

(3) $\tan^{-1}\left(\frac{2}{\pi}\right)$ (4) $\tan^{-1}\left(\frac{4}{\pi}\right)$

34. If the current through an inductor of inductance L is given by $I = I_0 \sin\omega t$, then the voltage across inductor will be :-

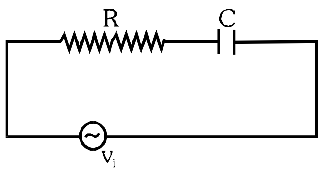
(1) $I_0\,\omega L \sin(\omega t - \pi/2)$ (2) $I_0\,\omega L \sin(\omega t + \pi/2)$

(3) $I_0\,\omega L \sin(\omega t - \pi)$ (4) None of these

35. There is a 5 Ω resistance in an A.C., circuit. Inductance of 0.1 H is connected with it in series. If equation of A.C. e.m.f. is 5 sin 50 t then the phase difference between current and e.m.f. is :-

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) 0

36. A 50 Hz a.c. source of 20 volts is connected across R and C as shown in figure below. The voltage across R is 12 volts. The voltage across C is



(1) 8 V

(2) 16V

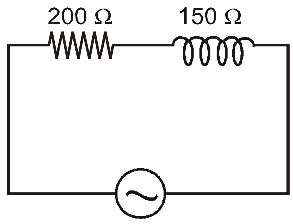
(3) 10 V

(4) Not possible to determine unless values of R and C are given

37. 200 Ω resistance and 1H inductance are connected in series with an A.C. circuit. The frequency of the source is $\frac{200}{2\pi}$ Hz. Then phase difference in between V and I will be :-

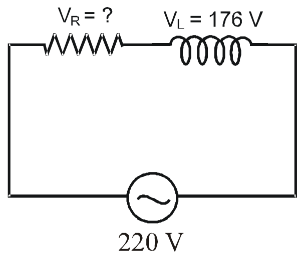
(1) 30° (2) 60° (3) 45° (4) 90°

38. Impedance of the following circuit will be :



(1) 150Ω (2) 200Ω (3) 250Ω (4) 340Ω

39. In showing figure find V_R :



(1) 132V (2) 396V

(3) 185 V (4) $\sqrt{220\times176}$V

40. If alternating current of 60 Hz frequency is flowing through inductance of L=1 mH and drop in ΔV_L is 0.6 V then alternating current :-

(1) $\frac{1}{\pi}$A (2) $\frac{5}{\pi}$A (3) $\frac{50}{\pi}$A (4) $\frac{20}{\pi}$A

LCR SERIES CIRCUIT, RESONANCE

41. An inductance of 1*mH*, a condenser of 10μ*F* and a resistance of 50Ω are connected in series. The reactance of inductor and condensers are same. The reactance of either of them will be :-

(1) 100 Ω (2) 30 Ω (3) 3.2 Ω (4) 10 Ω

42. L, C and R represent physical quantities inductance, capacitance and resistance respectively. The combination representing dimension of frequency is

(1) LC (2) $(LC)^{-1/2}$ (3) $\left(\frac{L}{C}\right)^{-1/2}$ (4) $\frac{C}{L}$

43. A circuit contains R, L and C connected in series with an A. C. source. The values of the reactances for inductor and capacitor are 200Ω and 600Ω respectively and the impedance of the circuit is Z_1. What happens to the impedance of the same circuit if the values of the reactances are interchanged:-

(1) The impedance will remain unchanged

(2) The impedance will increase

(3) The impedance will decrease

(4) Information insufficient

44. When V = 100 sinωt is applied across a series (R-L-C) circuit, At resonance the current in resistance (R=100 Ω) is $i = i_0 \sin\omega t$, then power dissipation in circuit is:-

(1) 50 W (2) 100 W

(3) 25 W (4) Can't be calculated

45. At resosnance in a series LCR circuit, which of the following statements is true:-

(1) Current in the circuit is maximum and phase difference between E and I is $\pi/2$

(2) Current in the circuit is maximum and phase difference between E and I is zero

(3) Voltage is maximum and phase difference between E and I is $\pi/2$

(4) Current is minimum and phase difference between E and I is zero

46. An alternating voltage is connected in series with a resistance r and an inductance L. If the potential drop across the resistance is 200 volt and across the inductance is 150 volt, the applied voltage:

(1) 350 volt (2) 250 volt

(3) 500 volt (4) 300 volt

47. For a series R-L-C circuit :-

(a) Voltage across L and C are differ by π

(b) Current through L and R are in same phase

(c) Voltage across R and L differ by $\pi/2$

(d) Voltage across L and current through C are differ by $\pi/2$

(1) a, b, c (2) b, c, d (3) c, d, a (4) All

48. A series R - L - C (R = 10 Ω, X_L = 20 Ω, Xc = 20 Ω) circuit is supplied by V = 10 sin ωt volt then power dissipation in circuit is :-

(1) Zero (2) 10 watt

(3) 5 watt (4) 2.5 watt

49. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50Hz. It should be connected to a capacitance of :

(1) 2×10^{-6} F (2) 3×10^{-6} F

(3) 10^{-4} F (4) 10^{-6} F

50. In a series resonant R-L-C circuit, if L is increased by 25% and C is decreased by 20%, then the resonant frequency will :

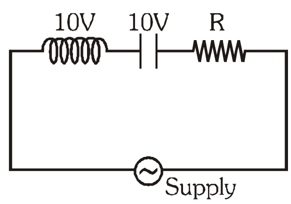
(1) Increases by 10% (2) Decreases by 10%

(3) Remain unchanged (4) Increases by 2.5%

51. The value of quality factor is :-

(1) $\frac{\omega L}{R}$ (2) $\frac{\omega}{RC}$ (3) $\sqrt{LC}$ (4) L/R

52. If value of R is changed, then :-



(1) Voltage across L remains same

(2) Voltage across C remains same

(3) Voltage across LC combination remains same

(4) Voltage across LC combination changes

53. In a series LCR circuit voltage across resister, inductor and capacitor are 1V, 3V and 2V respectively. At the instant t when the source voltage is given by :

$V=V_0 \cos \omega t$, the current in the circuit will be :

(1) $I = I_0 \cos\left(\omega t+\frac{\pi}{4}\right)$ (2) $I=I_0 \cos\left(\omega t-\frac{\pi}{4}\right)$

(3) $I = I_0 \cos\left(\omega t+\frac{\pi}{3}\right)$ (4) $I=I_0 \cos\left(\omega t-\frac{\pi}{3}\right)$

54. In an AC Circuit decrease in impedance with increase in frequency is indicates that circuit has/have :-

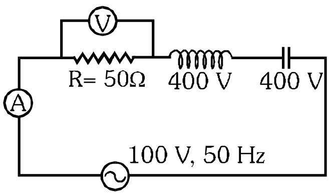
(1) Only resistance

(2) Resistance & inductance.

(3) Resistance & capacitance

(4) Resistance, capacitance & inductance.

55. In given LCR circuit, the voltage across the terminals of a resistance & current will be–



(1) 400V, 2A (2) 800V, 2A

(3) 100V, 2A (4) 100V, 4A

56. Phase of current in LCR circuit –

(1) Is in the phase of potential

(2) Leading from the phase of potential

(3) Lagging from the phase of potential

(4) Before resonance frequency, leading from the phase of potential and after resonance frequency, lagging from the phase of potential

57. In LCR circuit, the voltage across the terminals of a resistance, inductance & capacitance are 40V, 30V & 60V, then the voltage across the main source will be –

(1) 130 volt (2) 100 volt

(3) 70 volt (4) 50 volt

58. For an alternating current of frequency $\frac{500}{\pi}$ Hz in L-C-R series circuit with L = 1H, C = 1 μF, R = 100Ω, impedance is :-

(1) 100 Ω (2) $100\sqrt{\pi}\,\Omega$

(3) $100\sqrt{2\pi}\,\Omega$ (4) 100 π Ω

POWER IN AC CIRCUIT

59. A sinusoidal A.C. current flows through a resistor of resistance *R*. If the peak current is I_p, then the power dissipated is :-

(1) $I_p^2 R\cos\theta$ (2) $\frac{1}{2} I_p^2 R$

(3) $\frac{4}{\pi} I_p^2 R$ (4) $\frac{1}{\pi^2} I_p^2 R$

60. An AC circuit draws 5A at 160 V and the power consumption is 600 W. Then the power factor is:-

(1) 1 (2) 0.75 (3) 0.50 (4) Zero

61. Which is not correct for average power P at resonance :

(1) $P=I_{rms} V_{rms}$ (2) $P = \frac{V}{\sqrt{2}}\frac{I}{\sqrt{2}}$

(3) P=VI (4) $P=I^2_{rms} R$

62. In an A.C. circuit inductance, capacitance and resistance are connected. If the effective voltage across inductance is V_L , across capacitance is V_c and across resistance is V_R, then the total effective value of voltage is :

(1) $V_R+V_L+V_c$ (2) $V_R+V_L-V_c$

(3) $\sqrt{V_R^2+(V_L-V_C)^2}$ (4) $\sqrt{V_R^2-(V_L-V_C)^2}$

63. In an a.c. circuit V and I are given by

V = 100 sin (100 t) volts

I = 100 sin (100t + π/3) mA

The power dissipated in the circuit is

(1) 10^4 watt (2) 10 watt

(3) 2.5 watt (4) 5.0 watt

64. For a series LCR circuit the power loss at resonance is :-

(1) $\dfrac{V^2}{\left[\omega L-\dfrac{1}{\omega C}\right]}$ (2) $I^2L\omega$

(3) I^2R (4) $\dfrac{V^2}{C\omega}$

65. In an alternating circuit applied voltage and flowing current are $E = E_0 \sin\omega t$ and $I = I_0 \sin(\omega t+\pi/2)$ respectively. Then the power consumed in the circuit will be :

(1) Zero (2) $E_0I_0/2$

(3) $E_0I_0/\sqrt{2}$ (4) $E_0I_0/4$

66. In which of the following case power factor will be negligible :–

(1) Inductance and resistance both high

(2) Inductance and resistance both low.

(3) Low resistance and high inductance

(4) High resistance and low inductance

67. If $V = 100 \sin 100t$ volt, and

$I = 100 \sin(100t + \frac{\pi}{6})$ A. then find the watt less power in watt :-

(1) 10^4 (2) 10^3

(3) 10^2 (4) 2.5×10^3

68. An A.C. supply gives 30V r.m.s. which passes through a 10Ω resistance. The power dissipated in it is :-

(1) $90\sqrt{2}$ W (2) 90W

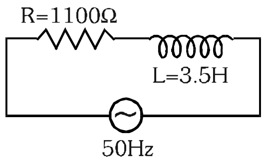
(3) $45\sqrt{2}$ W (4) 45 W

69. An inductor of inductance L and resistor of resistance R are joined in series and connected by a source of frequency ω. Power dissipated in the circuit is :-

(1) $\dfrac{(R^2+\omega^2L^2)}{V}$ (2) $\dfrac{V^2R}{(R^2+\omega^2L^2)}$

(3) $\dfrac{V}{(R^2+\omega^2L^2)}$ (4) $\dfrac{\sqrt{R^2+\omega^2L^2}}{V^2}$

70. For given circuit the power factor is :



(1) 0 (2) 1/2

(3) $1/\sqrt{2}$ (4) None of these

71. In a purely capacitive circuit average power dissipated in the circuit is -

(1) $V_{rms}\, I_{rms}$

(2) Depends on capacitance

(3) Infinite

(4) Zero

72. Energy loss in pure capacitance in A.C. circuit is

(1) $\frac{1}{2}CV^2$ (2) CV

(3) $\frac{1}{4}CV^2$ (4) Zero

73. Power dissipated in pure inductance will be :

(1) $\dfrac{LI^2}{2}$ (2) $2LI^2$

(3) $\dfrac{LI^2}{4}$ (4) Zero

74. The power factor of L-R circuit is :

(1) $\dfrac{\omega L}{R}$ (2) $\dfrac{R}{\sqrt{(\omega L)^2+R^2}}$

(3) ωLR (4) $\sqrt{\omega LR}$

75. If alternating current of rms value 'a' flows through resistance R then power loss in resistance is :

(1) Zero (2) a^2R

(3) $\dfrac{a^2R}{2}$ (4) $2a^2R$

76. Which of the following device in alternating circuit provides maximum power :-

(1) Only capacitor

(2) Capacitor and resistor

(3) Only inductor

(4) Only resistor

LC Oscillation

77. Comparing the L–C oscillations with the oscillations of a spring–block system (force constant of spring = k and mass of block = m), the physical quantity mk is similar to :–

(1) CL (2) $\frac{1}{CL}$

(3) $\frac{C}{L}$ (4) $\frac{L}{C}$

78. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is-

(1) Q/2 (2) $Q/\sqrt{3}$

(3) $Q/\sqrt{2}$ (4) Q

79. A fully charged capacitor C with intial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is :-

(1) $2\pi\sqrt{LC}$ (2) $\sqrt{LC}$

(3) $\pi\sqrt{LC}$ (4) $\frac{\pi}{4}\sqrt{LC}$

80. A LC circuit is in the state of resonance. if C = 0.1 μF and L = 0.25 henry. Neglecting ohmic resistance of circuit what is the frequency of oscillations

(1) 1007 Hz (2) 100 Hz

(3) 109 Hz (4) 500 Hz

81. A 60 μF capacitor is charged to 100 volts. This charged capacitor is connected across a 1.5 mH coil, so that LC oscillations occur. The maximum current in the coil is :-

(1) 1.5 A (2) 2 A

(3) 15 A (4) 20 A

EXERCISE-I (Conceptual Questions) **ANSWER KEY**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	2	4	3	2	3	4	1	4	3	2	1	3	1	3
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	3	2	2	1	2	1	2	1	3	4	4	3	3	2	3
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	2	2	2	2	3	2	3	3	1	2	4	2	1	1	2
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	4	3	4	3	1	3	2	3	3	4	4	1	2	2
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	3	3	3	3	1	3	4	2	2	3	4	4	4	2	2
Que.	76	77	78	79	80	81									
Ans.	4	4	3	4	1	4									

EXERCISE-II(Assertion & Reason) — Target AIIMS

Directions for Assertion & Reason questions

These questions consist of two statements each, printed as Assertion and Reason. While answering these Questions you are required to choose any one of the following four responses.

(A) If both Assertion & Reason are True & the Reason is a correct explanation of the Assertion.

(B) If both Assertion & Reason are True but Reason is not a correct explanation of the Assertion.

(C) If Assertion is True but the Reason is False.

(D) If both Assertion & Reason are false.

1. ***Assertion:-*** Power loss in ideal choke coil is zero.

Reason :- Ideal choke coil has zero resistance.

(1) A (2) B (3) C (4) D

2. ***Assertion:-*** Average power loss in series LC circuit circuit is always zero.

Reason :- Average values of voltage and current in A.C. is zero.

(1) A (2) B (3) C (4) D

3. ***Assertion :-*** Average of sinusoidal A.C. can never be zero for half cycle.

Reason :- Impedance given by inductance does not depends on frequency.

(1) A (2) B (3) C (4) D

4. ***Assertion :-*** The division are equally marked on the scale of A.C. ammeter.

Reason :- Heat produced is directly proportional to the current.

(1) A (2) B (3) C (4) D

5. ***Assertion :-*** The alternating current lags behind the e.m.f. by a phase angle of $\pi/2$, when A.C. flows through an inductor.

Reason :- The inductive reactance increases as the frequency of A.C. source decreases.

(1) A (2) B (3) C (4) D

6. ***Assertion :-*** Capacitor serves as a block for D.C.and offers an easy path to A.C.

Reason :- Capacitive reactance is inversely proportional to frequency.

(1) A (2) B (3) C (4) D

7. ***Assertion :-*** Choke coil is prefered over a resistor to adjust current in an A.C. circuit.

Reason :- Power factor for inductance is zero, so power loss is also zero.

(1) A (2) B (3) C (4) D

8. ***Assertion :-*** The resistance of a coil for direct current is 5 ohm. An alternating current is sent through it. The resistance will remain same.

Reason :- The resistance of a coil does not depend upon nature of current.

(1) A (2) B (3) C (4) D

9. ***Assertion :-*** When capacitive reactance is smaller then the inductive reactance in series LCR circuit, voltage leads the current.

Reason :- In series LCR circuit inductive reactance is always greater then capacitive reactance.

(1) A (2) B (3) C (4) D

10. ***Assertion :-*** In series RL ac circuit voltage leads the current.

Reason :- In series RC ac circuit current leads the voltage.

(1) A (2) B (3) C (4) D

11. ***Assertion :-*** An alternating current of frequency 50 Hz becomes zero, 100 times in one second.

Reason :- Alternating current changes direction and becomes zero twice in a cycle.

(1) A (2) B (3) C (4) D

12. ***Assertion :-*** An alternating current is more dangerous than a direct steady current.

Reason :- The frequency of the alternating current is injurious to the human body.

(1) A (2) B (3) C (4) D

13. ***Assertion :-*** A sinusoidal alternating current does not show any average magnetic effects.

Reason :- The average value of an alternating current is zero.

(1) A (2) B (3) C (4) D

14. ***Assertion :-*** The dc and ac both can be measured by a hot wire instrument.

Reason :- The hot wire instrument is based on the principle of heating effect of current.

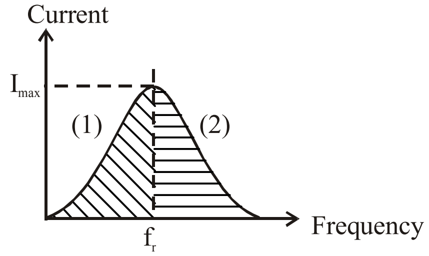
(1) A (2) B (3) C (4) D

15. ***Assertion :-*** In the series LCR circuit, the impedance is minimum at resonance.

Reason :- The currents in inductance and capacitance are same and voltage out of phase at resonance in series LCR circuit.

(1) A (2) B (3) C (4) D

16. For series RLC as circuit graph between current and frequency of voltage source is shown in figure.



Assertion :- Nature of circuit in region (1) is capacitive while in region (2) is inductive.

Reason :- Power factor of circuit in region (1) is negative and in region (2) is positive.

(1) A (2) B (3) C (4) D

17. ***Assertion :-*** In series LCR circuit phase difference between current and voltage is never zero.

Reason :- Voltage and current are never in phase. **[AIIMS 2015]**

(1) A (2) B (3) C (4) D

18. ***Assertion :-*** Capacitor is used to minimize power loss. **[AIIMS 2018]**

Reason :- A capacitor alters the phase voltage.

(1) A (2) B (3) C (4) D

19. ***Assertion :-*** In LR Circuit ohm's law is not obeyed at every instant. **[AIIMS 2018]**

Reason :- Current always lags from voltage.

(1) A (2) B (3) C (4) D

EXERCISE-II (Assertion & Reason) **ANSWER KEY**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	2	4	4	3	1	1	1	3	2	1	1	1	1	2
Que.	16	17	18	19											
Ans.	3	4	1	4											